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The option to stock volume ratio and future returns ☆

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ABSTRACT

We examine the information content of option and equity volumes when trade direction is unobserved. In a multimarket asymmetric information model, equity short-sale costs result in a negative relation between relative option volume and future firm value. In our empirical tests, firms in the lowest decile of the option to stock volume ratio (O/S) outperform the highest decile by 0.34% per week (19.3% annualized). Our model and empirics both indicate that O/S is a stronger signal when short-sale costs are high or option leverage is low. O/S also predicts future firm-specific earnings news, consistent with O/S reflecting private information.

1. Introduction

In recent decades, the availability of derivative securities has rapidly expanded. This expansion is not limited to equity options and now includes a vast array of securities ranging from currency options to credit default swaps. Derivatives contribute to price discovery because they allow traders to better align their strategies with the sign and magnitude of their information. The leverage in derivative securities, combined with this alignment, creates additional incentives to generate private information. In this way, trades in derivative markets may provide more refined and precise signals of the underlying asset's value than trades of the asset itself. Understanding how and why derivatives affect price discovery is therefore vital to understanding how information comes to be in asset prices.

This study focuses on the information content of trading volumes. Observed transactions play an important role in price discovery because order flow imbalances can reflect the sign and magnitude of private information. While market makers can observe these imbalances, most outside observers cannot, which makes the problem of inferring private information more complex. Techniques to empirically estimate order flow imbalances are computationally intensive, typically requiring the pairing of intraday trades and quotes. This problem is exacerbated when agents have access to multiple trading venues because the mapping between transactions and private information becomes more difficult to identify. In this paper, we address the inference problem of the outside

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observer by examining the information content of option and equity volumes when agents are privately informed but trade direction is unobserved.

We provide theoretical and empirical evidence that informed traders' private information is reflected in O/S, the ratio of total option market volume (aggregated across calls and puts) to total equity market volume. The O/S measure was first coined and studied by Roll, Schwartz, and Subrahmanyam (2010), whose findings suggest that cross-sectional and time-series variation in O/S could be driven by informed trade. As a natural extension of these findings, we examine the relation between O/S and future returns. Empirically, we find that contrasting publicly available totals of firm-specific option and equity volume portends directional prices changes, in particular that low O/S firms outperform the market while high O/S firms underperform. At the end of each week, we sort firms by O/S and compute the average return of a portfolio consisting of a short position in stocks with high O/S and a long position in stocks with low O/S. This portfolio provides an average risk-adjusted hedge return of 0.34% in the week following the formation date (19.3% annualized).

If option volume is concentrated among risky firms with higher return volatility, one might anticipate the opposite result, namely that firms with higher O/S earn higher future returns. While our finding is inconsistent with this risk-based explanation, we take several steps to mitigate concerns that exposure to other forms of risk (liquidity risk, for example) explains the O/S-return relation. First, we show that the relation holds after controlling for exposure to the three Fama-French and momentum factors. Second, we show that the predictive power of O/S for future returns is relatively short-lived. Strategy returns rapidly decline from 0.34% in the first week following portfolio formation and become statistically insignificant beyond the sixth week. Third, to mitigate concerns that our results are driven by static firm characteristics correlated with O/S and expected returns, we show that two measures of within-firm changes in O/S also predict future returns.

We argue that the negative relation between O/S and future returns is driven by short-sale costs in equity markets, which make option markets an attractive venue for traders with negative news. Motivated by this story, we model the capital allocation decision of privately informed traders who can trade in option and equity markets. Equity short-sale costs lead informed agents to trade options more frequently for negative signals than positive ones, thus predicting a negative relation between relative option volume and future equity value. An important innovation of our paper is that this relation does not require classifying trades as being buyer- versus seller-initiated. Instead, our theoretical predictions and empirical tests rely on publicly available volume totals.

Having established the negative cross-sectional relation between O/S and future returns, we next test our model's prediction that this relation is stronger when short-sale costs are high. As short-sale costs increase, informed traders are more likely to switch from equities to options for negative signals, which strengthens the O/S-return relation. We test this prediction using three different measures of short-sale costs. The first measure is derived from institutional ownership, as in Nagel (2005), and is available throughout our 1996–2010 sample window. We also use two direct measures of short-sale costs, transacted loan fees and available loan supply, from a proprietary database of institutional lending that is available on a monthly basis from 2002 through 2009. Across all three measures, we find that portfolio alphas associated with O/S are generally increasing in the cost of shorting, though the statistical significance of this pattern is mixed.

An additional empirical prediction arising from our model is that the O/S-return relation is weaker when option leverage is high. As option leverage increases, bid–ask spreads in options markets increase, which weakens the O/S-return relation because the bid–ask spread acts like a switching cost for traders considering the use of options to avoid short-sale costs. When option market bid–ask spreads are larger, fewer traders switch from equities to options for negative signals, and the O/S-return relation is therefore weaker. Empirically, we find that portfolio alphas associated with O/S are monotonically decreasing in option leverage.

It may be initially puzzling why we do not find a relation between the ratio of call to put volume and future returns. Our model demonstrates that O/S provides a clearer signal of private information than the ratio of call to put volume because call volume could be good news (if informed traders are buying) or bad news (if informed traders are selling), and put volume is similarly ambiguous. Thus, in the absence of information about the sign of each trade (i.e., buy vs. sell), O/S is an indication of the sign of private information while the ratio of call to put volume is not. Our model does, however, predict a positive relation between call–put volume differences and future return skewness because informed traders buy calls (puts) for extremely good (bad) news and sell calls (puts) for moderately bad (good) news. Consistent with this prediction, we show empirically that the ratio of call volume to put volume predicts return skewness in the subsequent week.

We also find that O/S predicts the sign and magnitude of earnings surprises, standardized unexplained earnings, and abnormal returns at quarterly earnings announcements in the following week. These tests show that the same O/S measure we use to predict weekly returns also contains information about future earnings announcements that occur in the subsequent week. This is consistent with O/S reflecting private information that is incorporated into equity prices following a subsequent public disclosure of the news.

The rest of the paper is organized as follows. We begin in Section 2 by discussing our results in the context of existing literature. We model the multimarket price discovery process and formalize the equilibrium strategy of informed traders in Section 3. In Section 4, we describe the data, methodology, empirical results, and robustness checks. Finally, we present results pertaining to quarterly earnings announcements in Section 5 and conclude in Section 6.

2. Relation to literature

The two immediate antecedents of our work are Easley, O'Hara, and Srinivas (1998), hereafter referred to as EOS, and Roll, Schwartz, and Subrahmanyam (2010), hereafter RSS. EOS contains a multimarket equilibrium model wherein privately informed traders are allowed to trade in both option and equity markets.[1] The EOS model highlights conditions under which informed traders transact in both option and equity markets, and predicts that directional option volume signals private information not yet reflected in equity prices. Specifically, their model predicts that positive trades (i.e., buying calls and selling puts) are positive signals of equity value and that negative trades (i.e., selling calls and buying puts) are negative signals of equity value. An interesting but otherwise unexplored empirical finding in EOS is that negative option market activity carries greater predictive power for future price changes. EOS comment on this finding in the following excerpt:

> An interesting feature of our results is the asymmetry between the negative- and positive-position effects … suggesting that options markets may be relatively more attractive venues for traders acting on "bad" news. An often-conjectured role for options markets is to provide a means of avoiding short-sales constraints in equity markets … Our results support this conjecture, suggesting a greater complexity to the mechanism through which negative information is impounded into stock prices [p. 458].

We provide a formal means of understanding their finding by introducing short-sale costs into a microstructure framework with asymmetric information. Like EOS, informed agents trade with a risk-neutral market maker, and can buy or sell shares of stock, buy or sell calls, or buy or sell puts. Unlike EOS, we impose short-sale costs that play a central role in determining which assets informed traders choose to trade. It is comparatively cheaper to capitalize on bearish private signals in option markets because traders can buy puts or sell calls, and in both cases they can create new option contracts without first borrowing them from a third party. In our model's equilibrium, the costs associated with short-selling make informed traders more likely to use options for bad signals than for good ones and, as a result, high O/S indicates negative private information and low O/S indicates positive private information.

Like EOS, we solve a static model and therefore need the additional assumption that some friction prevents equity prices from immediately reflecting the information in option volumes in order for the model's prediction about the conditional mean equity value to translate into return predictability. Our main empirical prediction, that O/S is a negative cross-sectional signal of future returns, differs from EOS in that it can be tested empirically without signing the direction of trades. We predict and confirm that contrasting publicly available totals of firm-specific option and equity volume portends directional price changes.

Empirically, our study of the relation between O/S and future returns is a natural extension of the work in RSS, which introduces the option to stock volume ratio, and coins it O/S. The authors find substantial intertemporal and cross-sectional variation in O/S, and explain a significant part of this variation in a regression framework. In particular, O/S is increasing in firm size and implied volatility but decreasing in option bid–ask spreads and institutional holdings. Our results shed additional light on the variation in O/S by examining the theoretical determinants of relative option volume when a subset of market participants is privately informed, and the empirical relation between O/S and future returns. RSS also show that O/S in the days immediately prior to announcement predicts the magnitude of returns at earnings announcements, consistent with O/S reflecting traders' private information. Conditional on there being positive (negative) earnings news, they find that O/S predicts higher (lower) announcement returns (see Section 5 for more details). Our analysis builds upon this finding by demonstrating an unconditional predictive relation between the prior week's O/S and earnings surprises.

Another recent paper examining option volume is Roll, Schwartz, and Subrahmanyam (2009), which shows a positive cross-sectional relation between Tobin's q and unscaled option volume. The authors interpret this as evidence that liquid option markets increase firm value because they help complete markets and generate informed trade. Our model and empirical tests support this intuition by demonstrating that option markets are an attractive venue for informed traders.

The results of this paper also relate to the literature on price discovery and information flow in multiple markets.[2] Pan and Poteshman (2006) use proprietary Chicago Board Options Exchange (CBOE) option market data and provide strong evidence of informed trading in option markets. The authors find that sorting stocks by the amount of newly initiated positions in puts relative to calls foreshadows future returns but they conclude the predictability is not due to market inefficiencies and instead reflects the fact that their volume measure is not publicly observable. A key innovation of our paper is demonstrating that publicly available, non-directional volume totals predict future returns. Similarly, Cremers and Weinbaum (2010) and Zhang, Zhao, and Xing (2010) find that publicly available asymmetries in implied volatility across calls and puts predict future returns.

Prior research establishes that equity volume, the denominator of our primary return predictor O/S, is useful

[1] The authors point out that asymmetric information violates the assumptions underlying complete markets and, therefore, the option trading process is not redundant. Consistent with this idea, Bakshi, Cao, and Chen (2000) find that Standard & Poors (S&P) 500 call options frequently move in the opposite direction of equity prices.

[2] Whether option markets lead equity markets or vice versa remains an open question. Anthony (1988) examines the interrelation of stock and option volumes and finds that call-option activity predicts volume in the underlying equity with a one-day lag. Similar findings are reported in Manaster and Rendleman (1982). In contrast, Stephan and Whaley (1990) find no evidence that options lead equities.

by itself in predicting future returns, though the direction depends on the way volume is measured (see, e.g., Gervais, Kaniel, and Mingelgren, 2001; Lee and Swaminathan, 2000; Brennan, Chordia, and Subrahmanyam, 1998). We decouple O/S into separate measures of equity and option volume and show that past option volume is negatively related to future returns incremental to past equity volume. Other extant work uses equity volume as a conditioning variable for examining the relation between past and future returns. Specifically, Lee and Swaminathan (2000) show that high (low) volume winners (losers) experience faster momentum reversals, and Llorente, Michaely, Saar, and Wang (2002) show that the relation between equity volume and return autocorrelation changes sign depending on the amount of informed trading for a given equity.

During the 2008 financial crisis, the U.S. Securities and Exchange Commission (SEC) banned short sales for 797 "financial" stocks, providing an interesting case study of the impact of short-sale costs on options markets. Both Battalio and Schultz (2011) and Grundy, Lim, and Verwijmeren (in press) find that option market spreads increased and option market volume decreased for firms subject to the ban relative to those exempt from it. A key component of our model is that option markets serve as an alternative venue for negative news when shorting is costly, and at first glance, the 2008 episode contradicts this premise. However, as emphasized in Battalio and Schultz (2011), the short-sale ban also imposed costs on option market makers who short equity, making it more difficult for them to hedge when selling puts or buying calls. In our model, increasing costs for market makers who write puts or buy calls will increase spreads and *decrease* volume in option markets, while banning shorts will increase both spreads and volume in options markets. Our model therefore suggests we interpret the decrease in option market volume during the 2008 ban as a result of the added costs of shorting for market makers outweighing the relocation of trades stemming from negative information to option markets.

In modelling the relation between short-sale costs and informed trading, our paper is also related to Diamond and Verrecchia (1987), which models the impact of short-sale constraints on the speed of adjustment of security prices to private information when informed traders only have access to equity markets. In their model, short-sale constraints cause some informed parties with negative information not to trade. Thus, the absence of trade in their model is a negative signal of future firm value. In our model, trading options is an alternative to abstaining from trade when the cost of shorting is high. As a result, a high option volume ratio, rather than the absence of trade, reflects negative private information.

3. The model

We present a model of informed trading in both equity and options markets in the presence of short-sale costs. Informed traders build a portfolio by trading sequentially with a competitive, risk-neutral market maker. A key feature of our model is that traders must pay a lending fee to a third party when shorting stock. Because it is costly to trade on bad news in the stock market, in equilibrium the mean equity value conditional on an option trade is lower than the mean equity value conditional on a stock trade.

There are three tradable assets in the model: an equity, a call option, and a put option. The stock liquidates for $\tilde{V}$ at time $t=2$ in the future. The value of $\tilde{V}$ is unknown prior to $t=2$, but it is common knowledge that

$$\tilde{V} = \mu + \tilde{\epsilon} + \tilde{\eta}, \tag{1}$$

where μ is the exogenous mean equity value, and $\tilde{\epsilon}$ and $\tilde{\eta}$ are independent, normally distributed shocks with zero mean and variances σ_ϵ^2 and σ_η^2. The call and put are both struck at μ, and both expire at time $t=2$. We focus on the case of European options with a single strike price because our aim is to model the choice between trading options and trading equities, as opposed to the choice amongst options of different strikes or times to expiration. We use μ as a strike price so that calls and puts have the same leverage.

Trade occurs at time $t=1$, at which point a fraction α of the traders (henceforth "informed traders") know the realization of $\tilde{\epsilon}$ but the remaining traders, and the market maker, do not. The distribution of $\tilde{V}$ conditional on the information that $\tilde{\epsilon} = \epsilon$ is

$$\tilde{V} \,|\, (\tilde{\epsilon} = \epsilon) \sim \mathrm{N}(\mu + \epsilon, \sigma_\eta^2). \tag{2}$$

Informed traders are risk-neutral, and therefore value the stock, call, and put using

$$\mathrm{E}(\tilde{V} \,|\, \tilde{\epsilon} = \epsilon) = \mu + \epsilon, \tag{3}$$

$$\mathrm{E}(\tilde{C} \,|\, \tilde{\epsilon} = \epsilon) = \Phi\left(\frac{\epsilon}{\sigma_\eta}\right)\epsilon + \phi\left(\frac{\epsilon}{\sigma_\eta}\right)\sigma_\eta, \tag{4}$$

$$\mathrm{E}(\tilde{P} \,|\, \tilde{\epsilon} = \epsilon) = -\Phi\left(\frac{-\epsilon}{\sigma_\eta}\right)\epsilon + \phi\left(\frac{\epsilon}{\sigma_\eta}\right)\sigma_\eta, \tag{5}$$

respectively, where Φ is the standard normal's cumulative distribution function, ϕ is its probability distribution function, and $\tilde{C}$ and $\tilde{P}$ are the values of the call and put at $t=2$.

We require that each trade be in exactly one type of asset, resulting in six possible trades: buy or sell stock, buy or sell calls, and buy or sell puts. At equilibrium prices, the informed traders have a strict preference among the assets for all signals other than six cutoff points.[3] A fraction $1-\alpha$ of the traders are uninformed and trade for reasons outside the model, possibly a desire for liquidity, the need to hedge other investments or human capital, or a false belief that they have information. Regardless of their motivation, uninformed traders choose among the same possible transactions as the informed traders, with fractions q_1, q_2, q_3, q_4, q_5, and q_6 choosing to

[3] Allowing trades in bundles of multiple assets (for example, one call and two shares) complicates the analysis without changing our results or providing additional insight. Bundles serve as "intermediate" portfolios used by the informed trader upon receiving a signal near their indifference point between the two bundled assets. As long as bundle trades that include a short position in equities require traders to pay short-sale costs, our model still predicts that equity (option) volume reflects positive (negative) private information.

buy stock, sell stock, buy calls, sell calls, buy puts, and sell puts, respectively, where $\sum_{i=1}^{6} q_i = 1$.

A competitive and risk-neutral market maker posts bid and ask prices for all three assets that result in zero expected profit for each trade.[4] For notation, we write a_s, b_s, a_c, b_c, a_p, and b_p for the ask and bid prices of the stock, call, and put, respectively. As in EOS and Glosten and Milgrom (1985), trades occur sequentially and at fixed order sizes: γ shares of stock and θ options contracts. Unlike EOS, we assume throughout that $\theta > 2\gamma$ so that options trades allow more exposure to the underlying than stock trades, an intuition expressed in Black (1975) as well as EOS and RSS (see Appendix B for details).

A critical new ingredient in our model is a short-sale cost paid by the trader to a third party who lends them the shares. The fee is a fraction $\rho > 0$ of the total amount shorted γb_s. The lender is able to charge such a fee because they have some market power, or because there is some counterparty risk. No such fee exists when writing options because there is no need to find a contract to borrow—the market maker can create a new contract. The parameter ρ can also represent a reduced form of any cost to shorting stock; for example, recall risk or the indirect costs described in Nagel (2005). Regardless of ρ's interpretation, the market maker pays γb_s for γ shares, but the trader only nets $\gamma b_s(1-\rho)$ from the transaction.[5]

3.1. Equilibrium

An equilibrium in our model consists of an optimal trading strategy for informed traders as a function of their signal, and bid–ask prices and quantities that yield zero expected profit for the market maker. In equilibrium, informed traders use the following cutoff strategy $f(\epsilon)$ that maps the range of possible signals to the space of possible trades:

$$f(\epsilon) = \begin{cases} \text{buy puts} & \text{for } \epsilon \le k_1, \\ \text{sell stock} & \text{for } \epsilon \in (k_1,k_2], \\ \text{sell calls} & \text{for } \epsilon \in (k_2,k_3], \\ \text{make no trade} & \text{for } \epsilon \in (k_3,k_4], \\ \text{sell puts} & \text{for } \epsilon \in (k_4,k_5], \\ \text{buy stock} & \text{for } \epsilon \in (k_5,k_6], \\ \text{buy calls} & \text{for } \epsilon > k_6. \end{cases} \quad (6)$$

For extremely good or bad signals, informed traders buy options despite large bid–ask spreads in these markets because they provide greater leverage. The bid–ask spreads make options unattractive for weaker signals, and so informed traders trade equities instead. For even weaker good or bad signals, however, informed traders value the stock near its unconditional mean and therefore cannot profitably trade stock. However, they value the options well below their unconditional mean because extreme outcomes occur with lower probabilities, and therefore sell options. If bid prices are below informed traders' valuation of both a put and a call for a given signal, informed traders choose not to trade. The cutoff points k_i arise endogenously in equilibrium and are chosen so that informed traders strictly prefer writing puts for all $\epsilon < k_1$, selling stock for all $k_1 < \epsilon < k_2$, etc. Some regions can be empty in equilibrium, meaning $k_i = k_{i+1}$ for some i. The addition of short-sale costs shrinks the region of signals for which informed traders short stock $(k_1,k_2]$.

The bid and ask prices for each asset (a_s, b_s, a_c, b_c, a_p, and b_p) and the informed traders' cutoff points k_i are the 12 equilibrium parameters. Together they satisfy 12 equations, presented fully in Appendix A, which assure that the market maker's expected profit is zero for each trade and that informed traders are indifferent between the two relevant trades at each cutoff point.

3.2. Results and empirical predictions

Due to the nonlinearity of the simultaneous equations, no closed-form solution for the equilibrium parameters is available. We derive our results and empirical predictions from the simultaneous equations. Our focus is on the information content of trading volumes when there are short-sale costs, so we assume throughout that $\rho > 0$. Proofs are in Appendix C.

Result 1. When each asset is equally likely to be bought or sold by an uninformed trader, the stock is worth less conditional on an option trade than it is conditional on a stock trade.

Empirical Prediction 1. Option volume, scaled by volume in the underlying equity, is negatively related to future stock returns.

The main result is that an option trade is bad news for the value of the stock and a stock trade is good news, which differs from EOS in that the conditioning variable is the location of trade rather than the direction of trade. Option volume reflects bad news because informed traders use stocks more frequently to trade on good news than bad due to the short-sale cost. Therefore, the expected equity value conditional on an option trade is lower than the unconditional mean, which is in turn lower than the expectation conditional on a stock trade. Result 1 requires that uninformed traders buy and sell each asset with equal probability, but holds regardless of how uninformed traders distribute their demand across the different assets; for example, uninformed traders may trade equities more frequently than options.

To translate Result 1 into an empirical prediction, we consider the implications of our static model in a multi-period setting. If equity markets fully incorporate the information revealed through options trading into their valuations, stock prices will immediately reflect the new conditional expectation of $\tilde{V}$ after each option trade.

[4] In the model, additional market makers have no impact as long as they are risk-neutral and competitive. The return predictability evidence in this paper suggests there is some segmentation between option and equity markets, perhaps because they have different market makers.

[5] It is important for our argument that option market makers do not pay the short-sale cost ρ in the course of hedging their position, and therefore embed the short-sale cost in option prices. In reality, option market makers have access to cheaper shorting than ordinary investors, and therefore the option-embedded short-sale cost is smaller than the actual short-sale cost in equity markets.

Otherwise, stock prices do not fully reflect the information content of options trades for the time between when the informed option market trading occurs and when the information becomes public through another channel. In this case, there will be a negative relation between option volume and subsequent returns until the public release of the information. Empirical Prediction 1 is, therefore, a joint hypothesis that (a) short-sale costs make O/S a negative cross-sectional predictor of future prices, and (b) some of the information in O/S reaches equities through other channels, such as earnings announcements, that occur after the observation of O/S.[6]

Our model makes no prediction about the overall volume in options and stocks together, only that option trades are bad news relative to stock trades. Our goal is to focus on informed traders' choice between equities and options, conditional on having a signal about the future value of a firm. Therefore, our predictive measure is the ratio of option volume to equity volume, rather than unscaled option volume.

Result 2. The disparity in conditional mean equity values between option and stock trades is weakly increasing in the short-sale cost ρ.

Empirical Prediction 2. The predictive power of relative option volume for future stock returns is increasing in the cost of shorting equity.

Although Empirical Prediction 1 does not rely on cross-sectional differences in short-sale costs, if such differences exist, our model predicts that option volume is a worse signal for high short-sale cost equities than low short-sale cost equities, but is still a valuable signal as long as short-sale costs exist.

Result 3. The disparity in conditional mean equity values between option and stock trades is weakly decreasing in the option's leverage $\lambda \equiv (\partial C/\partial S)S/C = \theta/2\gamma$.

Empirical Prediction 3. The predictive power of relative option volume for future stock returns is decreasing in the average Black-Scholes λ of options traded.

Result 3 may be surprising at first because leverage is usually an attractive feature of options. Indeed, in our model leverage allows an informed trader's investment to be more sensitive to their private information, and therefore the overall use of options by informed traders increases with leverage. However, this very attractiveness creates large bid–ask spreads in options markets, making it more expensive for informed traders to switch from trading equities to options to avoid the short-sale cost. Therefore, they make this switch for a smaller range of signals, which weakens the O/S-return relation.

Empirically, Result 3 suggests that volume in options markets with higher leverage provides a weaker signal than volume in options markets with lower leverage. For a measure of leverage, we use $\lambda = (\partial C/\partial S)S/C$, the elasticity of C with respect to S, reflecting the "bang-for-the-buck" notion of leverage. We show in Appendix B that $(\partial C/\partial S)S/C = \theta/2\gamma$ in our model. Empirically, we use the Black-Scholes λ because our model's λ does not account for different strike prices. Result 3 indicates there exists a spread between conditional means of $\tilde{V}$ regardless of the leverage λ, but that the spread is larger for smaller values of λ. Therefore, our empirical prediction is that O/S predicts returns for all levels of λ, but most strongly for low λ.

Result 4. Equity value has higher skewness conditional on a call trade than conditional on a put trade.[7]

Empirical Prediction 4. The ratio of call volume to put volume varies positively with the future skewness of stock returns.

Result 4 follows from the equilibrium trading strategy described in Section 3.1. Following the notation used to describe the informed trader's strategy in Eq. (6), skewness conditional on a put trade is low because, if informed, it reflects either moderately good news (i.e., $\epsilon \in (k_4, k_5]$) or extremely bad news (i.e., $\epsilon \leq k_1$). Similarly, skewness conditional on a call trade is high because, if informed, it reflects either moderately bad (i.e., $\epsilon \in (k_2, k_3]$) or extremely good news (i.e., $\epsilon > k_6$).

4. Empirical tests

The option data for this study come from the Ivy OptionMetrics database, which provides end-of-day summary statistics on all exchanged-listed options on U.S. equities. The summary statistics include option volume, quoted closing prices, and option Greeks. The OptionMetrics database, and hence the sample for this study, spans from 1996 through 2010. The final sample for this study is dictated by the intersection of OptionMetrics, Compustat Industrial Quarterly, and Center for Research in Security Prices (CRSP) Daily data. We restrict the sample to firm-weeks with at least 25 call and 25 put contracts traded to reduce measurement problems associated with illiquid option markets. We require each observation to have a minimum of six months of past weekly option and equity volumes because some of our analyses involve measuring firms' volumes relative to their historical averages. We also eliminate closed-end funds, real estate investment trusts, American depository receipts, and firms with a stock price below $1. The intersection of these databases and data restrictions results in 611,173 firm-weeks corresponding to approximately 730 calendar weeks and 1,660 unique firms per year.

[6] A common intuition is that call volume reflects good news and put volume reflects bad news. Eq. (6) demonstrates that this intuition does not hold in our setting because informed traders buy calls and sell puts for good news, and buy puts and sell calls for bad news. Therefore, unless trade direction is observable, it is unclear whether call (put) volume reflects good or bad news.

[7] Our proof of this result requires that the uninformed trader demand for each asset γ_i does not approach zero. If it did, markets would begin to fail and the skewness result can reverse.

For each firm i in week w, we sum the total option and equity volumes, denoted by $OPVOL_{i,w}$ and $EQVOL_{i,w}$, respectively. Specifically, $OPVOL_{i,w}$ equals the total volume in option contracts across all strikes for options expiring in the 30 trading days beginning five days after the trade date.[8] We report $EQVOL_{i,w}$ in round lots of 100 to make it more comparable to the quantity of option contracts that each pertain to 100 shares. We define the option to stock volume ratio, or $O/S_{i,w}$, as

$$O/S_{i,w} = \frac{OPVOL_{i,w}}{EQVOL_{i,w}}. \quad (7)$$

Panel A of Table 1 contains descriptive statistics of $O/S_{i,w}$ (hereafter O/S for notational simplicity) for each year in our sample. The sample size increases substantially over the 1996–2010 window. The number of firm-weeks increases from 29,426 in 1997 to 45,243 in 2010.[9] The remainder of Panel A presents descriptive statistics of O/S for each year of the sample. The sample mean of O/S is 5.77%, which indicates that there are roughly 17 times more equity round lots traded than option contracts with times to expiration between five and 35 trading days. O/S is positively skewed throughout the sample period due to a high concentration of relative option volume among a small subset of firms.

Panel B of Table 1 presents volume characteristics by deciles of O/S. Although low O/S firms tend to be smaller, our initial data requirement of 25 calls and 25 puts traded in a week tilts our sample toward larger and more liquid firms, which mitigates, but fails to eliminate, concerns that the O/S-return relation is attributable to transaction costs. The average market capitalization of firms exceeds \$2 billion in each O/S decile. VLC and VLP indicate the number of call and put contracts traded in a given week, respectively. Across all deciles of O/S, the number of call contracts traded exceeds the number of put contracts, which is consistent with calls being more liquid than puts. High O/S firms also tend to have higher levels of both option and equity volume, though equity volume changes much less across the O/S deciles. In our model, high O/S reflects negative private information, and hence our univariate trading strategy based on O/S consists of taking a short position in higher equity volume stocks (i.e., high O/S stocks) and a long position in lower equity volume stocks (i.e., low O/S stocks). This raises concerns that the predictive power of O/S could reflect compensation for taking positions in low liquidity firms. We attempt to mitigate these concerns in several ways, which are discussed in greater detail below. Panel B also presents firm characteristics by deciles of O/S. SIZE (LBM) equals the log of market capitalization (book-to-market) corresponding to firms' most recent quarterly earnings announcement. MOMEN equals firms' cumulative return measured over the prior six months. High O/S firms tend to be larger, have lower book-to-market (BM) ratios, and higher return momentum.

Panel A of Table 2 presents time-series factor regressions for each O/S decile using capital asset pricing model (CAPM), three-factor, and four-factor risk adjustments. To compute weekly O/S decile returns, we sort firms by O/S at the end of each week, skip one trading day, and compute the equal-weighted return for a portfolio of all firms in each decile over the following five trading days.[10] For example, when there are no trading holidays, we compute O/S from Monday through Friday of a given calendar week, skip the Friday-to-Monday return, and compute a weekly return from the close of markets on Monday to the close of markets on the following Monday.

To calculate four-factor portfolio alphas, we regress the weekly excess return corresponding to each O/S decile on the contemporaneous three Fama-French and momentum factors.[11] Specifically, we estimate three variants of the following regression for each O/S decile:

$$r_w^p - r_w^f = \alpha + \beta_1(r_w^{mkt} - r_w^f) + \beta_2 HML_w + \beta_3 SMB_w + \beta_4 UMD_w + \epsilon_w, \quad (8)$$

where r_w^p is the week w return on an equal-weighted portfolio of stocks in a given $O/S_{i,w-1}$ decile. We denote the risk-free rate as r_w^f and the market return as r_w^{mkt}. HML_w and SMB_w correspond to the weekly returns associated with high-minus-low market-to-book and small-minus-big strategies. Similarly, UMD_w equals the weekly return associated with a high-minus-low momentum strategy. The CAPM model omits all factors except for $r_w^{mkt} - r_w^f$ and the three-factor model omits UMD_w.

Our main result is that the intercepts from these regressions decrease with O/S, indicating that low O/S firms outperform high O/S firms. In the four-factor regression, we find that the portfolio of firms in the lowest O/S decile have a 0.19% alpha in the following week while the portfolio of firms in the highest O/S decile have a −0.15% alpha. The "1–10" row at the bottom of the table contains a statistical test for the difference of the low and high O/S decile portfolios, and shows that the 0.34% difference in the four-factor alphas are highly significant (t-statistic=5.00). The result is similar in statistical and economic magnitude for the CAPM and three-factor regressions, resulting in low-high alphas of 0.34% (t-statistic=4.20) and 0.30% (t-statistic=4.29), respectively. The final "(1+2)–(9+10)" row contains a statistical test for the difference between low and high O/S quintile portfolios,

[8] We exclude options expiring within five trading days to avoid measuring mechanical trading volume associated with option traders rolling forward to the next expiration date. The results are qualitatively unchanged if we include options with longer expirations. As an additional robustness check, we separate option volume into moneyness categories and find that at-the-money, in-the-money, and out-of-the-money option volumes all predict future returns once scaled by equity volume. The consistency of the O/S-return relation across moneyness categories mitigates concerns that our model omits critical determinants of the O/S-return relation by focusing on a single strike price.

[9] In our sample, 1996 has many fewer firm-week observations due to the requirement that six months of prior data be available for each firm.

[10] This restriction is important because of non-synchronous closing times across option and equity markets. Removing this restriction does not materially affect our results.

[11] We compute weekly factors to match our Monday close to subsequent Monday close time-frame by first compounding the returns for each of the size/BM and size/momentum portfolios and then computing the long-short return that defines the factors, as described on Ken French's Web site.

Table 1
Descriptive statistics by year.

Panel A provides sample size information and descriptive statistics of $O/S_{i,w}$ (shown as a percentage), where $O/S_{i,w}$ equals the ratio of option volume to equity volume of firm i in week w as outlined in Section 4. Panel B gives average firm characteristics by decile of O/S. The sample consists of 611,173 firm-weeks spanning 1996 through 2010. VLC (VLP) equals the total call (put) contract volume traded in a given week; each contract represents 100 shares. OPVOL equals the sum of VLC and VLP. EQVOL equals the total equity volume traded, in units of 100 shares. SIZE (LBM) equals the log of market capitalization (book-to-market) corresponding to firms' most recent quarterly earnings announcement. MOMEN equals firms' cumulative market-adjusted return measured over the six months prior to week w, in percent.

Panel A: Sample characteristics and O/S descriptive statistics by year

	Firms	Firm-weeks	MEAN	P25	MEDIAN	P75	SKEW
1996	1,020	12,006	6.260	2.163	4.359	8.494	5.822
1997	1,422	29,426	6.193	2.159	4.317	8.405	6.990
1998	1,655	32,970	5.636	1.866	3.768	7.381	5.803
1999	1,724	35,296	5.408	1.828	3.749	7.374	5.802
2000	1,733	40,696	5.024	1.873	3.738	7.057	68.868
2001	1,587	38,182	4.585	1.520	3.121	6.026	8.112
2002	1,533	36,087	3.835	1.283	2.765	5.619	4.507
2003	1,470	36,815	4.381	1.363	3.095	6.619	20.938
2004	1,590	41,062	5.425	1.615	3.782	8.280	9.801
2005	1,737	45,527	6.218	1.683	4.068	9.324	73.627
2006	1,848	52,299	7.329	1.941	4.786	10.775	26.834
2007	1,980	57,735	7.304	1.928	4.720	10.732	23.595
2008	1,914	57,035	6.249	1.523	3.722	8.648	8.684
2009	1,814	50,794	6.452	1.711	4.107	9.092	13.388
2010	1,870	45,243	6.322	1.542	3.792	8.643	9.521
ALL		611,173	5.775	1.733	3.859	8.165	19.486

Panel B: Firm characteristics by deciles of O/S

	VLC	VLP	OPVOL	EQVOL	SIZE	LBM	MOMEN
1 (Low)	228	124	353	74,095	7.734	0.375	0.461
2	479	249	728	66,965	7.555	0.359	2.147
3	827	439	1,266	74,415	7.542	0.350	2.972
4	1,342	727	2,069	85,184	7.594	0.342	3.754
5	2,080	1,160	3,240	97,036	7.671	0.332	4.300
6	3,390	1,924	5,314	116,808	7.788	0.323	5.189
7	5,264	3,103	8,368	136,539	7.933	0.312	5.751
8	8,072	4,993	13,064	156,023	8.091	0.301	6.878
9	12,145	7,791	19,936	164,785	8.190	0.293	7.942
10 (High)	25,197	15,807	41,003	148,488	8.128	0.273	11.058
High–low	24,968	15,683	40,651	74,393	0.394	−0.102	10.597

formed by combining the two lowest and two highest decile portfolios. The quintile strategy alphas attenuate relative to the decile strategy but remain economically and statistically significant for all three factor models.

As predicted by our model, in addition to high O/S indicating bad news, low O/S indicates good news: a portfolio of firms with low O/S has significantly positive alphas in the week after portfolio formation. In the context of our model, low relative option volume indicates good news because informed traders use equity more (and options less) frequently for positive signals than negative ones due to the equity short-sale costs.

Table 2 also presents the factor loadings (β coefficients from the estimates of Eq. (8)). We find that the low-high O/S strategy has a significantly negative loading on the market and UMD factors and a positive loading on the SMB factor. The negative market beta indicates that high O/S firms have more market exposure than low O/S firms, the opposite of what one would expect if the O/S-return relation reflects exposure to market risk. The remaining factor loadings confirm the univariate patterns shown in Table 1 in a multivariate setting: low O/S firms tend to be smaller firms with low book-to-market ratios and low momentum.

One potential concern with the results in Panel A of Table 2 is that some firms could have consistently higher O/S and lower average returns for reasons unrelated to our information story and not captured by the four-factor risk adjustment. To address this concern, Panels B and C of Table 2 reexamine our return predictability tests after sorting by within-firm changes in O/S. In Panel B, we sort firms by $\Delta O/S$, the change in O/S relative to a rolling average of past O/S for each firm. Specifically, we define $\Delta O/S$ as

$$\Delta O/S_{i,w} = \frac{O/S_{i,w} - \overline{O/S_i}}{\overline{O/S_i}}, \quad (9)$$

where $\overline{O/S_i}$ is the average $O/S_{i,w}$ for the firm over the prior six months. We sort the cross-section of firms by

Table 2

Factor regression results by deciles of O/S, ΔO/S, and ΩO/S.

Panel A presents factor regression results across deciles of $O/S_{i,w}$, where $O/S_{i,w}$ equals the ratio of option volume to equity volume of firm i in week w. Decile portfolios are formed at the conclusion of each week, ranging from 1 to 10 with the highest (lowest) values located in the 10th (1st) decile. The sample consists of 611,173 firm-weeks spanning 1996 through 2010. Portfolio returns are measured in week $w+1$ and regressed on three sets of contemporaneous risk factors: the excess market return (MKTRF); three Fama-French factors (MKTRF, SMB, and HML); and the three Fama-French and momentum factors (UMD). The intercept in this regression (INT) is the portfolio alpha. Panel B is defined analogously for ΔO/S, where ΔO/S equals the difference between O/S in the portfolio formation week and the average over the prior six months, scaled by this average. Panel C is defined analogously for ΩO/S, where ΩO/S is the percentile rank in the firm-specific time-series of O/S. All returns are shown as percentages, t-statistics are shown in parentheses.

Panel A: Factor regressions results across deciles of O/S

	CAPM		Three-factor				Four-factor				
	INT	MKTRF	INT	MKTRF	SMB	HML	INT	MKTRF	SMB	HML	UMD
1 (Low)	0.178	1.049	0.144	1.062	0.274	0.269	0.185	0.986	0.125	0.317	−0.257
	(2.12)	(35.30)	(1.77)	(35.80)	(5.51)	(4.86)	(2.37)	(33.02)	(2.45)	(5.94)	−(8.21)
2	0.014	1.135	−0.007	1.115	0.089	0.404	0.035	1.035	−0.068	0.455	−0.269
	(0.17)	(37.79)	−(0.09)	(37.33)	(1.77)	(7.25)	(0.45)	(34.57)	−(1.32)	(8.50)	−(8.57)
3	0.064	1.166	0.048	1.125	−0.011	0.510	0.083	1.059	−0.141	0.552	−0.223
	(0.72)	(37.13)	(0.57)	(36.69)	−(0.22)	(8.90)	(1.01)	(33.86)	−(2.64)	(9.87)	−(6.82)
4	−0.028	1.183	−0.034	1.128	−0.121	0.511	0.004	1.057	−0.261	0.556	−0.240
	−(0.31)	(36.99)	−(0.40)	(36.35)	−(2.33)	(8.82)	(0.04)	(33.53)	−(4.83)	(9.87)	−(7.25)
5	−0.072	1.253	−0.070	1.186	−0.217	0.514	−0.032	1.114	−0.360	0.560	−0.244
	−(0.78)	(38.21)	−(0.81)	(37.61)	−(4.10)	(8.72)	−(0.38)	(34.77)	−(6.56)	(9.78)	−(7.29)
6	−0.042	1.252	−0.043	1.182	−0.202	0.580	−0.010	1.119	−0.327	0.620	−0.214
	−(0.43)	(36.86)	−(0.49)	(36.45)	−(3.71)	(9.57)	−(0.11)	(33.63)	−(5.75)	(10.43)	−(6.15)
7	−0.029	1.301	−0.024	1.224	−0.273	0.554	0.005	1.170	−0.379	0.588	−0.181
	−(0.31)	(38.22)	−(0.27)	(37.84)	−(5.03)	(9.18)	(0.05)	(35.02)	−(6.63)	(9.85)	−(5.17)
8	−0.065	1.287	−0.052	1.202	−0.350	0.536	−0.032	1.166	−0.421	0.559	−0.123
	−(0.69)	(38.92)	−(0.61)	(38.86)	−(6.74)	(9.27)	−(0.38)	(36.14)	−(7.64)	(9.69)	−(3.63)
9	−0.101	1.244	−0.092	1.157	−0.333	0.598	−0.076	1.126	−0.394	0.618	−0.104
	−(1.09)	(37.76)	−(1.10)	(38.04)	−(6.52)	(10.53)	−(0.91)	(35.42)	−(7.25)	(10.87)	−(3.13)
10 (High)	−0.164	1.229	−0.158	1.147	−0.300	0.587	−0.153	1.137	−0.318	0.593	−0.031
	−(1.85)	(38.89)	−(1.97)	(39.29)	−(6.12)	(10.77)	−(1.90)	(37.06)	−(6.06)	(10.81)	−(0.98)
1–10	0.342	−0.180	0.302	−0.085	0.574	−0.318	0.338	−0.151	0.443	−0.275	−0.225
	(4.20)	−(6.22)	(4.29)	−(3.30)	(13.33)	−(6.63)	(5.00)	−(5.86)	(10.05)	−(5.97)	−(8.34)
(1+2)–(9+10)	0.229	−0.144	0.193	−0.063	0.498	−0.255	0.224	−0.121	0.384	−0.219	−0.195
	(3.45)	−(6.13)	(3.43)	−(3.08)	(14.46)	−(6.67)	(4.19)	−(5.90)	(11.00)	−(5.99)	−(9.10)

Panel B: Factor regressions results across deciles of ΔO/S

	CAPM		Three-factor				Four-factor				
	INT	MKTRF	INT	MKTRF	SMB	HML	INT	MKTRF	SMB	HML	UMD
1 (Low)	0.127	1.180	0.114	1.142	−0.027	0.447	0.163	1.050	−0.208	0.505	−0.310
	(1.37)	(35.82)	(1.27)	(34.93)	−(0.50)	(7.32)	(1.91)	(32.22)	−(3.73)	(8.67)	−(9.08)
2	0.018	1.252	0.010	1.200	−0.105	0.503	0.055	1.116	−0.270	0.556	−0.284
	(0.19)	(37.80)	(0.11)	(37.06)	−(1.93)	(8.32)	(0.64)	(34.28)	−(4.85)	(9.56)	−(8.33)
3	−0.003	1.270	−0.005	1.213	−0.160	0.486	0.031	1.145	−0.294	0.529	−0.229
	−(0.03)	(38.03)	−(0.05)	(37.18)	−(2.92)	(7.98)	(0.36)	(34.33)	−(5.15)	(8.88)	−(6.57)
4	−0.018	1.281	−0.013	1.209	−0.251	0.525	0.024	1.139	−0.388	0.570	−0.236
	−(0.19)	(38.23)	−(0.15)	(37.68)	−(4.66)	(8.77)	(0.28)	(34.83)	−(6.95)	(9.74)	−(6.88)
5	−0.022	1.257	−0.013	1.181	−0.301	0.502	0.019	1.121	−0.419	0.540	−0.202
	−(0.24)	(37.97)	−(0.15)	(37.45)	−(5.68)	(8.52)	(0.23)	(34.60)	−(7.56)	(9.32)	−(5.96)
6	−0.083	1.239	−0.078	1.170	−0.248	0.497	−0.052	1.123	−0.341	0.527	−0.160
	−(0.90)	(38.17)	−(0.91)	(37.55)	−(4.74)	(8.54)	−(0.62)	(34.77)	−(6.18)	(9.14)	−(4.74)

Panel B: Factor regressions results across deciles of ΔO/S

7	−0.089	1.226	−0.085	1.157	−0.243	0.505	−0.062	1.114	−0.327	0.532	−0.143
	−(1.00)	(38.70)	−(1.02)	(38.24)	−(4.79)	(8.94)	−(0.75)	(35.46)	−(6.08)	(9.47)	−(4.35)
8	−0.059	1.186	−0.060	1.120	−0.198	0.546	−0.040	1.083	−0.271	0.570	−0.127
	−(0.68)	(38.15)	−(0.74)	(37.94)	−(3.99)	(9.90)	−(0.50)	(35.22)	−(5.16)	(10.37)	−(3.93)
9	−0.025	1.153	−0.039	1.108	−0.043	0.522	−0.024	1.080	−0.097	0.539	−0.092
	−(0.30)	(39.19)	−(0.50)	(39.04)	−(0.91)	(9.85)	−(0.31)	(36.41)	−(1.91)	(10.17)	−(2.96)
10 (High)	−0.090	1.055	−0.120	1.030	0.132	0.531	−0.103	0.999	0.071	0.551	−0.104
	−(1.12)	(37.05)	−(1.59)	(37.68)	(2.86)	(10.40)	−(1.38)	(35.03)	(1.46)	(10.80)	−(3.47)
1–10	0.217	0.125	0.234	0.112	−0.159	−0.084	0.266	0.051	−0.279	−0.045	−0.206
	(3.38)	(5.48)	(3.68)	(4.82)	−(4.09)	−(1.94)	(4.38)	(2.18)	−(7.02)	−(1.09)	−(8.47)
(1+2)–(9+10)	0.129	0.112	0.140	0.102	−0.110	−0.052	0.172	0.043	−0.226	−0.014	−0.199
	(2.45)	(5.96)	(2.68)	(5.34)	−(3.43)	−(1.45)	(3.49)	(2.29)	−(7.02)	−(0.42)	−(10.10)

Panel C: Factor regressions results across deciles of ΩO/S

	CAPM		Three-factor				Four-factor				
	INT	MKTRF	INT	MKTRF	SMB	HML	INT	MKTRF	SMB	HML	UMD
1 (Low)	0.012	1.364	0.025	1.290	−0.318	0.456	0.096	1.156	−0.583	0.542	−0.454
	(0.10)	(32.13)	(0.22)	(30.76)	−(4.52)	(5.83)	(0.90)	(28.12)	−(8.29)	(7.38)	−(10.54)
2	0.077	1.321	0.085	1.260	−0.237	0.403	0.140	1.156	−0.442	0.469	−0.352
	(0.76)	(36.77)	(0.87)	(35.48)	−(3.98)	(6.07)	(1.52)	(32.82)	−(7.34)	(7.44)	−(9.54)
3	−0.009	1.286	−0.012	1.231	−0.149	0.468	0.038	1.138	−0.333	0.527	−0.316
	−(0.10)	(37.43)	−(0.13)	(36.42)	−(2.63)	(7.41)	(0.43)	(33.69)	−(5.77)	(8.73)	−(8.94)
4	0.039	1.267	0.040	1.209	−0.183	0.466	0.082	1.130	−0.339	0.517	−0.269
	(0.41)	(37.56)	(0.44)	(36.58)	−(3.29)	(7.56)	(0.94)	(33.77)	−(5.93)	(8.65)	−(7.67)
5	−0.015	1.192	−0.018	1.139	−0.146	0.463	0.013	1.081	−0.259	0.499	−0.194
	−(0.17)	(37.45)	−(0.21)	(36.58)	−(2.80)	(7.96)	(0.15)	(33.76)	−(4.74)	(8.73)	−(5.79)
6	−0.085	1.183	−0.088	1.131	−0.138	0.450	−0.062	1.082	−0.235	0.481	−0.166
	−(1.01)	(39.40)	−(1.10)	(38.65)	−(2.81)	(8.24)	−(0.79)	(35.80)	−(4.55)	(8.91)	−(5.25)
7	−0.034	1.172	−0.041	1.118	−0.115	0.511	−0.022	1.083	−0.185	0.533	−0.119
	−(0.40)	(38.69)	−(0.51)	(38.28)	−(2.35)	(9.36)	−(0.28)	(35.57)	−(3.55)	(9.80)	−(3.75)
8	−0.023	1.156	−0.034	1.107	−0.070	0.520	−0.021	1.082	−0.119	0.536	−0.085
	−(0.29)	(40.65)	−(0.46)	(40.67)	−(1.54)	(10.24)	−(0.28)	(37.98)	−(2.45)	(10.54)	−(2.84)
9	0.028	1.130	0.017	1.084	−0.063	0.503	0.031	1.056	−0.116	0.520	−0.092
	(0.34)	(38.89)	(0.22)	(38.61)	−(1.33)	(9.60)	(0.41)	(36.00)	−(2.31)	(9.92)	−(2.98)
10 (High)	−0.099	1.104	−0.115	1.056	−0.032	0.570	−0.108	1.043	−0.057	0.578	−0.042
	−(1.22)	(38.68)	−(1.55)	(39.08)	−(0.71)	(11.31)	−(1.46)	(36.75)	−(1.17)	(11.40)	−(1.43)
1–10	0.111	0.260	0.140	0.234	−0.286	−0.114	0.205	0.112	−0.526	−0.037	−0.411
	(1.19)	(7.88)	(1.53)	(7.00)	−(5.11)	−(1.83)	(2.45)	(3.51)	−(9.62)	−(0.64)	−(12.28)
(1+2)–(9+10)	0.099	0.224	0.124	0.204	−0.234	−0.110	0.177	0.104	−0.431	−0.047	−0.338
	(1.40)	(8.87)	(1.77)	(8.00)	−(5.48)	−(2.32)	(2.82)	(4.33)	−(10.53)	−(1.09)	−(13.48)

ΔO/S in each calendar week. Firms in the lowest decile of ΔO/S earn a four-factor alpha of 0.16% per week (*t*-statistic = 1.91). Similarly, firms in the highest decile of ΔO/S earn -0.10% per week (*t*-statistic = -1.38). The ΔO/S decile strategy produces an alpha of 0.27% per week (*t*-statistic = 4.38) and the ΔO/S quintile strategy produces an alpha of 0.17% per week (*t*-statistic = 3.49). The ΔO/S factor loadings are closer to zero than those for O/S, consistent with our change-based specification mitigating the influence of persistent firm characteristics that arise when sorting firms by the level of O/S. The two loadings that remain significant are SMB, which switches signs from positive in Panel A to negative in Panel B, and UMD, which remains strongly negative.

As an alternative means of calculating abnormal levels of O/S, Panel C of Table 2 estimates factor-adjusted portfolio returns after sorting firms based on within-firm variation in O/S. We sort each firm-week into an ΩO/S decile by ranking it relative to the firm's O/S time-series over the past six months (26 weeks). For example, firms in the highest ΩO/S decile have O/S above the 90th percentile of their own O/S distribution measured over the past six months. Given the large swings in O/S documented in RSS, we use six months of data to ensure that the firm's reference distribution has a sufficiently large number of observations to capture meaningful firm-specific variation in O/S. Our choice of six months is in line with Lee and Swaminathan (2000), Llorente, Michaely, Saar, and Wang (2002), Barber and Odean (2008), and Sanders and Zdanowicz (1992) that all use periods of six to 12 months to establish baseline levels of firm-specific volume. We find that the use of a longer reference window, such as one year, produces qualitatively similar results but reduces the number of observations available for our analyses. We also find similar results when using a shorter reference window, such as the 10 weeks used in Gervais, Kaniel, and Mingelgrin (2001), however doing so reduces the robustness of ΩO/S in predicting future returns.

The benefit of ΩO/S relative to ΔO/S is that it relies on firms' own rolling distribution to assess abnormal levels of O/S and thus can be calculated for a single firm without reference to the cross-sectional distribution. The cost of not referencing the cross-sectional distribution is that ΩO/S is more sensitive to market-wide changes in option or equity volumes that are unrelated to firm-specific private information. Because ΩO/S relies on pure time-series sorts, unlike the cross-sectional sorts in Panels A and B, there is no guarantee that we have an equal number of firms in each decile of ΩO/S in a given week. Regardless of the number of firms in each ΩO/S bin, we compute the weekly return of an equal-weighted portfolio of all constituent firms. We find that a portfolio consisting of a long position in the lowest decile of ΩO/S firms and a short position in highest decile of ΩO/S firms earns a four-factor alpha of 0.21% (*t*-statistic = 2.45), while a quintile strategy produces a four-factor alpha of 0.18% (*t*-statistic = 2.82). The consistency of return predictability across O/S, ΔO/S, and ΩO/S mitigates concerns that the O/S-return relation reflects compensation for a static form of risk.

Our main analyses focus on the relation between O/S and weekly returns. We chose a weekly horizon, rather than daily or monthly, to balance competing concerns. Although our model does not formally define the length of a given period, the endogenous determination of bid–ask spreads is intuitively linked to short horizons, for example the intraday volumes in EOS and the daily O/S in RSS. On the other hand, shorter horizons are subject to the concern that the pattern of predictable returns is attributable to portfolio rebalancing costs. While we present results here pertaining to weekly observations of O/S and returns, untabulated results demonstrate that our inferences are unchanged when conducting the analysis using daily or monthly sampling frequencies.[12]

Table 3 presents summary statistics from weekly Fama-MacBeth regressions where the dependent variable is the firm's return during the week after observing O/S, denoted by RET(1). Columns 1 through 3 of Panel A contain the results of regressing RET(1) on deciles of O/S. For example, in column 1 the O/S coefficient is -0.026, indicating that firms in the highest O/S decile outperform firms in the lowest decile by an average of 0.23% ($=-0.026\times 9$) per week. The O/S coefficient has a corresponding *t*-statistic of -3.99, where standard errors are computed across weekly coefficient estimates as in Fama and MacBeth (1973). Columns 1 through 7 demonstrate that the relation between O/S and RET(1) is robust to controlling for MOMEN, log market capitalization (SIZE), and log book-to-market (LBM). Columns 2 through 7 also control for the Amihud (2002) illiquidity ratio, AMIHUD, defined as the ratio of absolute returns to total dollar volume where higher values indicate lower liquidity, and vice versa. AMIHUD is measured on a daily basis and then averaged over the six months prior to portfolio formation. Columns 3 through 6 include returns in the portfolio formation week, RET(0), to control for the possibility of weekly return reversals. Consistent with the results in Jegadeesh (1990) and Lehmann (1990), the RET(0) coefficient is significantly negative, indicating a negative relation between returns in weeks $w-1$ and w. Across columns 1 through 3, the O/S coefficient is significantly negative, with the coefficients and *t*-statistics remaining stable across specifications.

Column 4 of Panel A contains regression results where O/S is decoupled into numerator and denominator, option volume (OPVOL) and equity volume (EQVOL). Because EQVOL and OPVOL are highly correlated with SIZE, our regression analysis instead uses changes in EQVOL and OPVOL, denoted by ΔEQVOL and ΔOPVOL. Following Eq. (9), the "Δ" version of each variable is the level of the variable less the average of the variable over the prior

[12] The main exception pertains to the concentration of the O/S-return relation among firms with high short-sale costs. Across all three horizons, we find that O/S strategy alphas are increasing in short-sale costs. However, while this result is statistically significant for the monthly and weekly horizons, it is not for the daily horizon. The absence of this effect at daily horizons is consistent with short-sale costs reflecting market frictions that prevent the information content of O/S from being reflected in equity prices in an immediate fashion. Untabulated results available from the authors upon request.

Table 3
Fama-MacBeth multivariate regressions.

Panel A presents Fama-MacBeth regression results from regressing RET(1) on deciles of O/S, ΔOPVOL, and ΔEQVOL. The sample consists of 611,173 firm-weeks spanning 1996 through 2010. RET(1) is the firm's return in the first week following the observation of $O/S_{i,w}$, the ratio of option volume to equity volume of firm *i* in week *w*. $OPVOL_{i,w}$ equals the total option volume of firm *i* in week *w*. $EQVOL_{i,w}$ is defined analogously for equity volume. ΔOPVOL is the difference between OPVOL in the observation week and the average OPVOL over the prior six months, scaled by this average. ΔEQVOL is defined analogously. Decile portfolios are formed at the conclusion of each week. Deciles range from 1 to 10 with the highest (lowest) values located in the 10th (1st) decile. RET(0) is the market-adjusted return in the portfolio formation week. MOMEN equals the cumulative market-adjusted returns measured over the prior six months. SIZE is the log of market capitalization of the firm and LBM is the log of the firm's book-to-market ratio measured at the firm's last quarterly announcement date. AMIHUD is the Amihud illiquidity ratio of firm *i* in week *w*. Panel B repeats this analysis using ΔO/S, the difference between O/S in the observation week and the average O/S over the prior six months, scaled by this average. In Panel C, ΩO/S is the percentile rank in the firm-specific time-series measured relative to the distribution of the firm's O/S over the past six months. ΩOPVOL and ΩEQVOL are analogously defined for OPVOL and EQVOL. Standard errors are computed across weekly coefficient estimates, following Fama and MacBeth (1973). The resulting *t*-statistics are shown in parentheses. The notations ***, **, and * indicate the coefficient is significant at the 1%, 5%, and 10% level, respectively. All returns are shown as percentages.

Panel A: Fama-MacBeth regressions of RET(1) on O/S

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	−0.507	−0.297	−0.355	−0.331	−0.355	−0.368	−0.312
	(−1.13)	(−0.59)	(−0.74)	(−0.69)	(−0.74)	(−0.77)	(−0.62)
Decile(O/S)	−0.026***	−0.025***	−0.022***	–	−0.028***	−0.019***	−0.022***
	(−3.99)	(−3.64)	(−3.32)	–	(−3.36)	(−2.88)	(−3.22)
Decile(ΔOPVOL)	–	–	–	−0.011**	0.013**	–	–
	–	–	–	(−2.38)	(1.98)	–	–
Decile(ΔEQVOL)	–	–	–	0.030***	–	0.023***	0.022***
	–	–	–	(4.99)	–	(3.99)	(3.65)
RET(0)	–	–	−0.015***	−0.014***	−0.015***	−0.014***	–
	–	–	(−3.39)	(−3.27)	(−3.38)	(−3.29)	–
MOMEN	0.003**	0.003**	0.003*	0.002*	0.002*	0.002*	0.003*
	(2.05)	(2.14)	(1.93)	(1.74)	(1.81)	(1.71)	(1.91)
SIZE	0.037	0.023	0.026	0.011	0.024	0.019	0.016
	(1.38)	(0.77)	(0.91)	(0.41)	(0.85)	(0.65)	(0.53)
LBM	0.137	0.141	0.159	0.228	0.146	0.180	0.169
	(0.79)	(0.82)	(0.97)	(1.36)	(0.90)	(1.11)	(1.00)
AMIHUD		−0.008*	−0.007*	−0.007*	−0.007*	−0.007*	−0.008*
		(−1.93)	(−1.71)	(−1.87)	(−1.77)	(−1.67)	(−1.85)
Adj-R^2 (%)	4.694	5.032	5.810	5.783	5.967	5.975	5.210

Panel B: Fama-MacBeth regressions of RET(1) on ΔO/S

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	−0.458	−0.225	−0.294	−0.331	−0.254	−0.308	−0.241
	(−0.99)	(−0.45)	(−0.61)	(−0.69)	(−0.53)	(−0.64)	(−0.47)
Decile(ΔO/S)	−0.020***	−0.019***	−0.013***	–	−0.042***	−0.011***	−0.017***
	(−4.36)	(−4.20)	(−3.20)	–	(−4.51)	(−2.71)	(−3.77)
Decile(ΔOPVOL)	–	–	–	−0.011**	0.036***	–	–
	–	–	–	(−2.38)	(3.53)	–	–
Decile(ΔEQVOL)	–	–	–	0.030***	–	0.025***	0.024***
	–	–	–	(4.99)	–	(4.18)	(3.84)
RET(0)	–	–	−0.015***	−0.014***	−0.015***	−0.014***	–
	–	–	(−3.37)	(−3.27)	(−3.35)	(−3.26)	–
MOMEN	0.003**	0.003**	0.003**	0.002*	0.003*	0.002*	0.003**
	(2.16)	(2.23)	(2.01)	(1.74)	(1.83)	(1.77)	(2.00)
SIZE	0.030	0.015	0.019	0.011	0.014	0.011	0.008
	(1.17)	(0.54)	(0.68)	(0.41)	(0.50)	(0.41)	(0.29)
LBM	0.206	0.207	0.214	0.228	0.222	0.229	0.228
	(1.16)	(1.17)	(1.27)	(1.36)	(1.32)	(1.37)	(1.31)
AMIHUD	–	−0.008**	−0.008*	−0.007*	−0.008*	−0.007*	−0.008**
	–	(−2.11)	(−1.90)	(−1.87)	(−1.91)	(−1.86)	(−2.02)
Adj-R^2 (%)	4.506	4.822	5.606	5.783	5.778	5.780	5.009

Panel C: Fama-MacBeth regressions of RET(1) on ΩO/S

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	−0.424	−0.224	−0.274	−0.364	−0.243	−0.338	−0.264
	(−0.90)	(−0.44)	(−0.56)	(−0.75)	(−0.49)	(−0.69)	(−0.51)
Decile(ΩO/S)	−0.016***	−0.015**	−0.010*	–	−0.033***	−0.010*	−0.016***
	(−2.64)	(−2.51)	(−1.85)	–	(−3.09)	(−1.76)	(−2.73)

Table 3 (*continued*)

Panel C: Fama-MacBeth regressions of RET(1) on ΩO/S

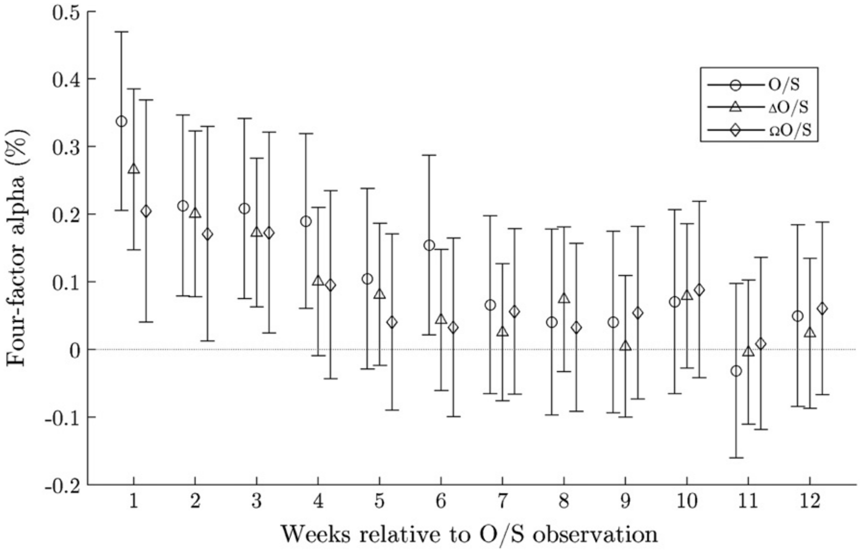
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Decile(ΩOPVOL)	–	–	–	−0.011*	0.029***	–	–
	–	–	–	(−1.76)	(2.60)	–	–
Decile(ΩEQVOL)	–	–	–	0.027***	–	0.022***	0.021***
	–	–	–	(4.06)	–	(3.36)	(3.12)
RET(0)	–	–	−0.014***	−0.013***	−0.013***	−0.013***	–
	–	–	(−3.22)	(−3.06)	(−3.07)	(−3.08)	–
MOMEN	0.003**	0.003**	0.003**	0.003*	0.003**	0.003*	0.003**
	(2.29)	(2.36)	(2.10)	(1.88)	(1.97)	(1.93)	(2.19)
SIZE	0.028	0.015	0.017	0.012	0.011	0.011	0.008
	(1.06)	(0.52)	(0.62)	(0.42)	(0.39)	(0.41)	(0.27)
LBM	0.209	0.210	0.211	0.232	0.227	0.234	0.234
	(1.20)	(1.21)	(1.27)	(1.40)	(1.38)	(1.42)	(1.35)
AMIHUD	–	−0.008*	−0.007*	−0.006	−0.007*	−0.006	−0.008*
	–	(−1.90)	(−1.74)	(−1.55)	(−1.66)	(−1.54)	(−1.91)
Adj-R^2 (%)	4.603	4.923	5.691	5.842	5.901	5.854	5.071

six months, all scaled by that average. Column 4 demonstrates that both the numerator and denominator contribute to predictability: the coefficient corresponding to deciles of ΔOPVOL is −0.011 (t-statistic = −2.38) and the coefficient corresponding to deciles of ΔEQVOL is 0.030 (t-statistic = 4.99). This is consistent with our model's prediction that high option volume reflects negative private information and high equity volume reflects positive private information, once controlling for both volume measures. The positive ΔEQVOL coefficient is also consistent with Gervais, Kaniel, and Mingelgrin (2001), which argues that abnormal volume garners additional visibility and therefore predicts higher future returns. Column 5 demonstrates that O/S remains negatively related to future returns after controlling for ΔOPVOL, but the ΔOPVOL coefficient is significantly positive, indicating that innovations in a firm's OPVOL are positive predictors of future returns after controlling for a firm's O/S. Finally, comparing the O/S coefficients in columns 3 and 6 or columns 2 and 7 shows that the O/S-return relation is relatively unaffected by controlling for equity volume. Taken together, the results in Panel A of Table 3 demonstrate a robust negative association between O/S and future equity returns, distinct from weekly return reversals, the pricing of liquidity, and the relation between equity market volume and future returns.

Panel B of Table 3 repeats the Fama-MacBeth regressions in Panel A but with ΔO/S replacing O/S. The main result from Panel B is that ΔO/S is negatively associated with future returns across all regression specifications, each controlling for a different combination of momentum, size, book-to-market, liquidity, short-term reversal, ΔOPVOL, and ΔEQVOL. Panel C of Table 3 repeats the Fama-MacBeth regressions using ΩO/S, ΩOPVOL, and ΩEQVOL and yields results that are qualitatively identical to the findings in Panel B. ΩO/S, ΩOPVOL, and ΩEQVOL rely on within-firm variation to measure how the underlying variable ranks relative to the firm's historical distribution. For example, the decile of ΩOPVOL reflects a firm-week's rank relative to the firm's OPVOL distribution over the prior six months. In both Panels B and C, comparing columns 2 and 7 or columns 3 and 6, we find that controlling for equity volume does not significantly affect the magnitude of the O/S-return relation regardless of whether we control for RET(0). However, comparing columns 2 and 3 or columns 6 and 7, we find that controlling for RET(0) reduces the magnitude of the ΔO/S and ΩO/S coefficients regardless of whether ΔEQVOL is a regressor. Across all specifications, ΔO/S and ΩO/S are significant negative predictors of future returns, suggesting that within-firm variation in O/S reflects the direction of informed trade.

Having established a robust relation between O/S, ΔO/S, and ΩO/S and future returns, we next examine the duration of return predictability associated with each measure. Fig. 1 shows the alphas from strategies with progressively longer delays between the observation of the O/S signal and the weekly return in question. For example, the O/S strategy with a four-week lag sorts firms by O/S measured four weeks prior to the realized return window. This results in a weekly return series that we use to compute four-factor alphas as in Eq. (8). Fig. 1 repeats this exercise with lags of 1–12 weeks, across all three measures: O/S, ΔO/S, and ΩO/S. The top graph shows weekly alphas and their 95% confidence interval. The bottom graph shows cumulative alphas.

The top graph in Fig. 1 shows that the return predictability associated with O/S is relatively short-lived, decreasing sharply but remaining statistically significant at the 5% level in the four weeks following portfolio formation. The significant O/S-return relation disappears after the sixth week following portfolio formation, which is inconsistent with the O/S-return relation reflecting a static dimension of risk correlated with O/S. The persistence of return predictability suggests that it takes multiple weeks for the information content of O/S to become fully reflected in equity prices. Like O/S, both ΔO/S and ΩO/S show patterns of return predictability that are short-lived. Repeating this analysis at the 1% significance level, we again find that return predictability associated with O/S and ΔO/S persists for four and three weeks, respectively (results untabulated). We also find that ΩO/S does not predict future returns at the 1% level at any horizon, consistent with the finding in Table 2 that



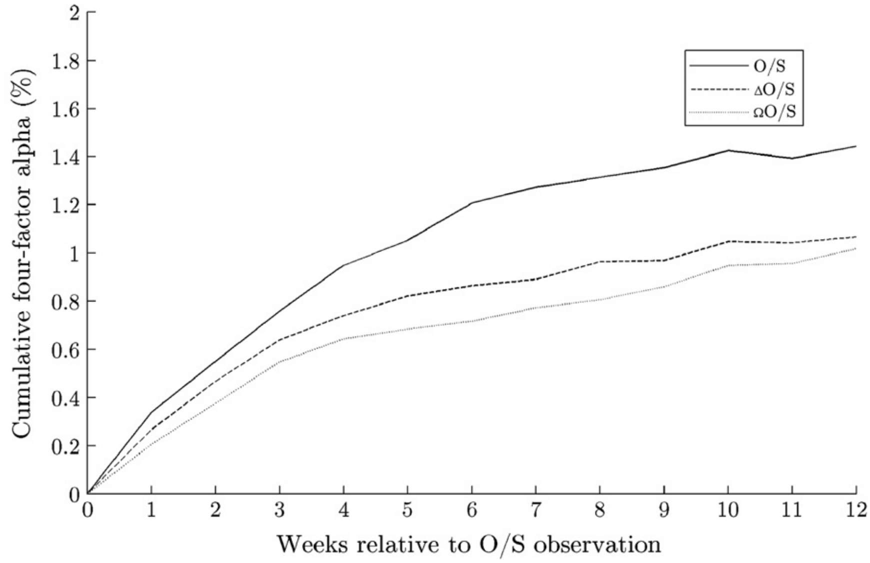


Fig. 1. Persistence of O/S-return relation. This figure presents the alphas associated with a portfolio that combines an equal-weighted long position in the lowest decile with an equal-weighted short position in the highest decile of each relative option volume signal. The top graph shows weekly alphas, where the surrounding error bars represent the 95% confidence interval. The bottom graph shows cumulative alphas. The portfolio is formed *K* weeks after the observation of the signal, where *K* ranges from 1 to 12. $O/S_{i,w}$ equals the ratio of option volume to equity volume of firm *i* in week *w*. ΔO/S equals the difference between O/S in the portfolio formation week and the average over the prior six months, scaled by this average. ΩO/S equals the percentile rank in the firm-specific time-series of O/S. Alphas are the intercept in a time-series regression of weekly strategy returns on contemporaneous weekly factor returns for the three Fama-French and momentum factors. The sample consists of 611,173 firm-weeks spanning 1996 through 2010. Alphas are shown as percentages.

ΩO/S has the lowest predictive power for future returns among the three O/S measures. Unlike O/S and ΔO/S, the ΩO/S measure does not reference the cross-sectional distribution when assigning firms to portfolios. Thus, the weaker predictive power associated with ΩO/S is consistent with the measure being more sensitive to market-wide variation in option or equity volumes that are unrelated to private information. The bottom graph in Fig. 1 shows that cumulative 12-week alphas range from 0.9% to 1.5%, where O/S outperforms ΔO/S and ΔO/S outperforms ΩO/S on a cumulative basis across all durations.

Table 4 presents tests of Empirical Prediction 2, that the predictive power of O/S for future returns is increasing in short-sale costs. Our first measure of firm-specific short-sale costs, following Nagel (2005), is the level of residual institutional ownership $RI_{i,q}$. We define $RI_{i,q}$ as the percentage of shares held by institutions for firm *i* in quarter *q*, adjusted for size in cross-sectional regressions. Specifically, $RI_{i,q}$ equals the residual $\epsilon_{i,q}$ from the following regression:

$$\text{logit}(INST_{i,q}) = \log\left(\frac{INST_{i,q}}{1-INST_{i,q}}\right) = \alpha_q + \beta_{1,q} SIZE_{i,q} + \beta_{2,q}(SIZE_{i,q}^2) + \epsilon_{i,q}, \quad (10)$$

Table 4
Strategy alphas sorted by short-sale costs.

This table presents alphas for portfolios double-sorted by one of three different short-sale cost measures (RI, LF, and LS), and by one of three different relative option measures (O/S, ΔO/S, and ΩO/S, as defined in Table 2). RI (residual institutional ownership) is obtained from cross-sectional regressions as detailed in Nagel (2005). LF (loan fee) is the value-weighted average loan fee for institutional loans in the month prior to portfolio formation, and LS (loan supply) is the quantity of shares available for lending scaled by shares outstanding at the end of the month prior to portfolio formation. In Panel A (B), firms are sorted each week into quintiles (deciles) of each relative option volume signal and quintiles (terciles) of each short-sale cost measure, and returns are measured the following week. Within each short-sale cost portfolio, strategy alphas are computed for a long-short position in the extreme O/S portfolios using a time-series regression on the three Fama-French and momentum factors (factor loadings not reported). The main sample consists of 611,173 firm-weeks spanning 1996 through 2010, however, the loan fee and supply data are only available from June 2002 through 2009. All returns are shown as percentages, *t*-statistics are in parentheses.

Panel A: Quintile alphas by quintiles of short-sale costs

	O/S	ΔO/S	ΩO/S
RI(1): High short-sale costs	0.269	0.219	0.124
	(2.24)	(2.01)	(0.99)
RI(2)	0.319	0.174	0.313
	(3.40)	(1.75)	(2.71)
RI(3)	0.185	0.201	0.109
	(2.39)	(2.61)	(1.17)
RI(4)	0.227	0.242	0.154
	(2.67)	(2.89)	(1.70)
RI(5): Low short-sale costs	0.090	0.039	0.181
	(1.17)	(0.49)	(1.98)
High–low short-sale costs	0.179	0.180	−0.057
	(1.34)	(1.34)	−(0.38)
LF(1): Low short-sale costs	0.011	−0.151	0.098
	(0.10)	−(1.13)	(0.72)
LF(2)	−0.070	−0.099	−0.105
	−(0.64)	−(0.91)	−(0.87)
LF(3)	0.156	0.088	0.066
	(1.23)	(0.68)	(0.46)
LF(4)	0.358	0.229	0.429
	(2.85)	(2.05)	(2.92)
LF(5): High short-sale costs	0.273	0.189	0.157
	(2.06)	(1.28)	(0.90)
High–low short-sale costs	0.262	0.340	0.059
	(1.56)	(1.87)	(0.16)
LS(1): High short-sale costs	0.258	0.553	0.431
	(1.76)	(3.76)	(2.42)
LS(2)	0.065	0.001	−0.013
	(0.50)	(0.00)	−(0.09)
LS(3)	0.130	0.119	0.144
	(1.06)	(1.09)	(1.05)
LS(4)	−0.024	0.035	0.115
	−(0.21)	(0.31)	(0.89)
LS(5): Low short-sale costs	−0.192	0.246	0.317
	−(1.33)	(2.19)	(2.56)
High–low short-sale costs	0.449	0.306	0.114
	(2.53)	(1.65)	(0.63)

Panel B: Decile alphas by terciles of short-sale costs

	O/S	ΔO/S	ΩO/S
RI(1): High short-sale costs	0.471	0.366	0.271
	(3.97)	(3.34)	(2.07)
RI(2)	0.358	0.272	0.151
	(3.90)	(3.20)	(1.41)
RI(3): Low short-sale costs	0.183	0.209	0.259
	(2.07)	(2.32)	(2.38)
High–low short-sale costs	0.288	0.157	0.009
	(2.10)	(1.13)	(0.06)
LF(1): Low short-sale costs	0.038	−0.132	−0.029
	−(0.34)	−(1.11)	−(0.20)
LF(2)	0.123	0.213	0.218
	(0.84)	(1.46)	(1.19)

Table 4 *(continued)*

Panel B: Decile alphas by terciles of short-sale costs

	O/S	ΔO/S	ΩO/S
LF(3): High short-sale costs	0.309	0.456	0.378
	(2.19)	(2.98)	(1.91)
High–low short-sale costs	0.348	0.588	0.479
	(1.98)	(3.13)	(2.08)
LS(1): High short-sale costs	0.396	0.541	0.314
	(2.52)	(3.68)	(1.68)
LS(2)	−0.010	0.170	0.176
	−(0.07)	(1.46)	(1.19)
LS(3): Low short-sale costs	−0.184	0.258	0.323
	−(1.17)	(2.17)	(2.15)
High–low short-sale costs	0.580	0.283	0.014
	(2.78)	(1.55)	(0.07)

where $INST_{i,q}$ equals the fraction of shares outstanding held by institutions as reflected in the Thomson Financial Institutional Holdings (13F) database. We calculate quarterly holdings as the sum of stock holdings of all reporting institutions for each firm and quarter. Values of $INST_{i,q}$ are winsorized at 0.0001 and 0.9999. Low levels of $RI_{i,q}$ (hereafter referred to as RI) correspond to high short-sale costs because stock loans tend to be scarce and, hence, short-selling is more expensive when institutional ownership is low. We match RI to a given firm-week by requiring a three-month lag between the Thomson Financial report date and the first trading day of a given week.

The other two measures of short-sale costs are more direct, and rely on a proprietary data set of institutional lending provided to us by Data Explorers. Data Explorers aggregates information on institutional lending from several market participants including hedge funds, investment banks, and prime brokers.[13] Similar to the data sets used in D'Avolio (2002) and Geczy, Musto, and Reed (2002), this data set contains monthly institutional lending data on transacted loan fees and available loan supply. The sample period is June of 2002 through December of 2009, covering approximately half of our main sample period. From this data set, we derive our second measure of firm-specific short-sale costs: LF, the value-weighted average loan fee for institutional loans occurring in the calendar month prior to the portfolio formation date. Higher values of LF reflect higher short-sale costs because investors must pay the lending fee to obtain the shares necessary for shorting. The final measure of short-sale costs, LS, is the total quantity of shares available for lending, as a fraction of firms' total shares outstanding, at the conclusion of the calendar month prior to portfolio formation. Lower values of LS correspond to higher short-sale costs because investors must first locate lendable shares before implementing a short position.

Panel A presents alphas for portfolios double-sorted into short-sale cost quintiles and relative option volume quintiles, for each of the three short-sale cost measures. Within each short-sale cost quintile, we compute the four-factor alpha of a long-short strategy using extreme quintiles of each relative option volume measure (O/S, ΔO/S, and ΩO/S). For example, the entry in the RI(1) row corresponding to the O/S signal indicates that among firms in the lowest residual institutional ownership quintile, a strategy long firms in the lowest O/S quintile and short firms in the highest O/S quintile produces a weekly alpha of 0.269% (*t*-statistic = 2.24). The key tests of Empirical Prediction 2 are contained in the "High-low short-sale costs" rows, which examine differences in strategy alphas across extreme short-sale cost quintiles. The results in Panel A are mixed. Eight of the nine differences in strategy alphas across extreme short-sale cost portfolios are positive, indicating that O/S is a stronger signal for future returns when short-sale costs are high. However, only three of the nine are statistically significant at the 10% level, and only one of the nine is significant at the 5% level.

Panel B presents analogous results for strategies relying on extreme deciles of relative option volume (rather than quintiles), sorted by terciles (rather than quintiles) of short-sale costs. We analyze 10×3 sorts because the results in Table 2 indicate that decile strategies produce larger alphas than quintile strategies, and because the results in Panel A show that the near-extreme quintiles of short-sale costs (2 and 4) often contain strategy alphas of different signs than the extreme quintiles (1 and 5). For example, the ΩO/S alpha is 0.098 in the lowest LF quintile but −0.105 in the second-lowest LF quintile. The results in Panel B confirm that 10×3 sorts produce a clearer difference in strategy alphas across short-sale costs. All nine of the "High-low short-sale costs" alphas are positive, and five of the nine are significant at the 5% level.

Across both panels of Table 4, the differences in strategy alphas across the extreme short-sale cost portfolios are economically and statistically stronger for O/S than the change-based measures, ΔO/S and ΩO/S. One potential explanation is that ΔO/S and ΩO/S generate weaker strategy alphas compared to O/S (as illustrated in Table 3) and we therefore do not have the statistical power to distinguish alphas across short-sale cost extremes. Another potential explanation is that our change-based measures are themselves correlated with changes in

[13] See www.dataexplorers.comfor more details regarding the data.

short-sale costs. In untabulated results, we show that $\Delta O/S$ and $\Omega O/S$ are much more strongly positively correlated with the prior week's return than O/S. As argued in Geczy, Musto, and Reed (2002) and D'Avolio (2002), short-sale costs are a decreasing function of recent returns, implying that $\Delta O/S$ and $\Omega O/S$ could be correlated with recent changes in short-sale costs that our monthly and quarterly short-sale cost measures fail to detect.

To summarize, Table 4 provides mixed support for Empirical Prediction 2. Across three different measures of short-sale costs and the three relative option volume measures, in nearly all cases the portfolio alphas associated with option volume strategies are higher for firms with high short-sale costs. These differences are more often statistically significant for the O/S strategy, and for 10×3 sorts. Collectively, the results in Table 4 provide some evidence that informed traders use option markets more frequently when short-sale costs are high.

Consistent with Empirical Prediction 3, Table 5 demonstrates that the predictive power of relative option volume for future stock returns is strongest when option leverage is low. For each firm-week, leverage is defined as the open-interest-weighted average of $(\partial C/\partial S)S/C$, as provided by OptionMetrics, which we refer to as LM.[14] Panel A of Table 5 contains factor regression results for long-short O/S quintile portfolios across quintiles of LM, where firms are independently sorted by LM and O/S. The O/S alphas are monotonically decreasing across quintiles of LM, where the difference in portfolio alphas across the extreme LM quintiles is significant at the 1% level (t-statistic$=$4.42). The second and third columns of Panel A indicate that the $\Delta O/S$ and $\Omega O/S$ strategy alphas are also strongest among firms with low leverage, with both alpha differences also significant at the 1% level. Finally, similar to Table 4, Panel B repeats the analysis using deciles of relative option volume and terciles of option leverage. As in Panel A, portfolio alphas are concentrated among firms with low option leverage for all three option volume signals (t-statistics$=$4.97, 3.81, and 1.49). The results in Table 5 are consistent with informed traders moving a larger portion of their bets from shorting stock to trading options when leverage is low.

To address the possibility that the O/S-return relation is specific to only a subsample of our data, Fig. 2 presents annual cumulative returns to three long-short strategies assuming weekly portfolio rebalancing for each year in the sample. The first strategy consists of an equal-weighted long-short position in the extreme O/S deciles. We implement the long-short strategy each week and compound the weekly returns within each calendar year. The unconditional long-short strategy (shown in grey) results in positive returns in 13 out of 15 years, with a mean return of 21.81% and a standard deviation of 20.64%.[15] The second strategy takes long-short positions in extreme O/S deciles among firms in the bottom tercile of residual institutional ownership (RI), corresponding to firms with the highest short-sale costs. This strategy (shown in black) produces positive returns in 13 out of 15 years of the sample, while increasing the mean of the annual cumulative returns to 40.37%.[16] The third strategy corresponds to analogous long-short returns for firms in the lowest leverage (LM) tercile. Conditional upon being in the lowest LM tercile, the long-short O/S strategy (shown in white) results in positive hedge returns in 13 of 15 years while again increasing the mean return relative to the unconditional O/S strategy to 27.46%. Across all three strategies, returns in the later sample years 2002–2010 are smaller than those in the early sample years 1996–2001 but still remain consistently positive and economically significant. Together, the results of Fig. 2 demonstrate a robust association between O/S and future returns throughout our sample

Table 5
Option volume alphas sorted by leverage.

This table presents alphas for portfolios double-sorted by terciles of open-interest-weighted average leverage (LM) of firm i in week w, and by one of three different relative option measures (O/S, ΔO/S, and ΩO/S, as defined in Table 2). In Panel A (B), firms are sorted each week into quintiles (deciles) of each relative option volume signal and quintiles (terciles) of leverage, and returns are measured the following week. Within each LM portfolio, strategy alphas are computed for a long-short position in the extreme O/S portfolios using a time-series regression on the three Fama-French and momentum factors (factor loadings not reported). The sample consists of 611,173 firm-weeks spanning 1996 through 2010. All returns are shown as percentages, t-statistics are in parentheses.

Panel A: Quintile alphas by quintiles of leverage

	O/S	ΔO/S	ΩO/S
LM(1): Low leverage	0.581	0.476	0.433
	(4.67)	(4.41)	(3.59)
LM(2)	0.247	0.021	0.100
	(2.39)	(0.22)	(0.93)
LM(3)	0.095	0.076	−0.053
	(1.12)	(0.95)	−(0.61)
LM(4)	(0.10)	(0.07)	0.143
	(1.45)	(1.25)	(2.05)
LM(5): High leverage	−0.007	0.039	0.061
	−(0.15)	(0.86)	(1.21)
Low-high leverage	0.589	0.437	0.372
	(4.42)	(3.84)	(2.89)

Panel B: Decile strategy alphas by terciles of leverage

	O/S	ΔO/S	ΩO/S
LM(1): Low leverage	0.711	0.499	0.262
	(5.53)	(4.29)	(1.83)
LM(2)	0.259	0.236	0.207
	(2.72)	(2.80)	(1.92)
LM(3): High leverage	0.042	0.004	0.040
	(0.71)	(0.06)	(0.60)
Low–high leverage	0.669	0.496	0.221
	(4.97)	(3.81)	(1.49)

[14] The results are qualitatively similar when using volume-weighted average option leverage.

[15] A comparable strategy using $\Delta O/S$ results in average annual returns of 12.67%, and positive returns in 12 out of 15 sample years. Sorting by $\Omega O/S$ results in average annual returns of 7.87%, and positive returns in nine of 15 sample years (results untabulated).

[16] Because the long-short strategy results rely upon taking positions among equities with high short-sale costs, the reported results are not intended to reflect the actual returns achieved through implementation.

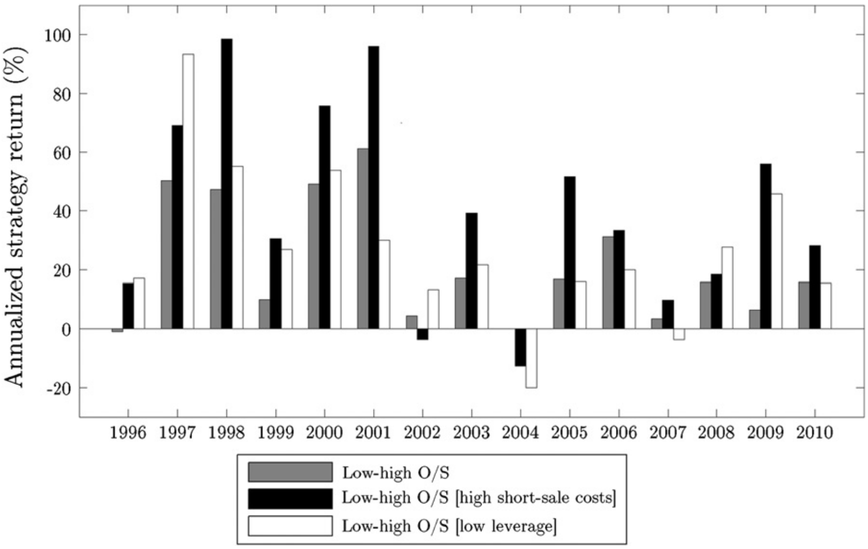


Fig. 2. Cumulative hedge returns by year. This figure presents cumulative annual unadjusted returns to three strategies assuming weekly portfolio rebalancing for each year in the sample. The first strategy (shown in grey) consists of an equal-weighted long position in the lowest $O/S_{i,w}$ decile together with an equal-weighted short position in the highest $O/S_{i,w}$ decile. $O/S_{i,w}$ equals the ratio of option volume to equity volume of firm *i* in week *w*. In addition to $O/S_{i,w}$ deciles, firms are independently sorted into terciles of residual institutional ownership and option leverage. The second strategy (shown in black) consists of a long-short $O/S_{i,w}$ position for firms in the lowest tercile of residual institutional ownership (RI). RI is obtained from cross-sectional regressions as detailed in Nagel (2005). The third strategy (shown in white) consists of a long-short $O/S_{i,w}$ position for firms in the lowest leverage (LM) tercile, where $LM_{i,w}$ is the open-interest-weighted average λ of firm *i* in week *w*. The sample consists of 611,173 firm-weeks spanning 1996 through 2010. All returns are shown as percentages.

Table 6
Future return skewness by deciles of call-put volume ratio.

The dependent variable in the table below is SKEW, defined as the cross-sectional skewness of weekly returns within a given portfolio in the week following portfolio formation. SKEW is calculated each calendar week and for each decile of $C/P_{i,w}$, where $C/P_{i,w}$ equals the ratio of total call volume to total put volume of firm *i* in week *w*. Decile portfolios are formed at the conclusion of each week. Deciles range from 1 to 10 with the highest (lowest) values located in the 10th (1st) decile. The sample consists of 611,173 firm-weeks spanning 1996 through 2010, from which we compute 7,330 decile-weeks. Year fixed effects are included and standard errors are clustered at the weekly level. The resulting *t*-statistics are shown in parentheses. The notations ***, **, and * indicate the coefficient is significant at the 1%, 5%, and 10% level, respectively.

Dep. variable:	SKEW	
	(1)	(2)
Intercept	0.233***	0.187***
	(4.59)	(3.68)
Decile(C/P)	0.030***	0.029***
	(5.89)	(5.69)
Lag(SKEW)	–	0.139***
	–	(10.91)
Adj-R^2 (%)	2.525	4.419

period, and that this association is stronger when short-sale costs are high or option leverage is low.

In addition to the above results pertaining to O/S, we also examine what the call to put volume ratio, C/P, tells us about future equity returns. Empirical Prediction 4 states that C/P is a positive predictor of future return skewness. The results of Table 6 confirm this prediction. For each firm-week, we compute C/P as

$$C/P_{i,w} = \frac{VLC_{i,w}}{VLP_{i,w}}, \tag{11}$$

where $VLC_{i,w}$ is the total call volume for firm *i* in week *w* and $VLP_{i,w}$ is defined analogously for puts. Firms are sorted into deciles based on C/P, where the tenth (first) decile corresponds to high (low) levels of call volume relative to put volume. For each calendar week, we calculate the cross-sectional skewness of the subsequent week's returns, RET(1), for each decile portfolio of C/P, which results in a panel data set of approximately 7,330 observations.

Table 6 contains the results of regressing cross-sectional skewness on the C/P decile rank. In column 1, the coefficient on the C/P decile rank is significantly positive (*t*-statistic=5.89), indicating that C/P is positively associated with future return skewness. Column 2 demonstrates that this relation remains significant after controlling for the lagged skewness of a given C/P decile. The evidence in Table 6 is consistent with our model's prediction that informed traders buy puts for extremely bad news, sell calls for moderate bad news, sell puts for moderate good news, and buy calls for extremely good news.

5. Additional analyses

Several existing studies examine the link between option market activity and earnings announcements. Skinner (1990) finds that the information content of earnings announcements declines following options listing, consistent with options facilitating informed trade prior to announcements. Amin and Lee (1997) find that open interest increases prior to announcements and possesses some predictive power for the sign of earnings news. RSS find that O/S significantly increases immediately prior to earnings announcements, suggesting that O/S reflects private information regarding earnings news. Consistent with this interpretation, they find that O/S positively predicts the absolute magnitude of earnings news and that the effect is more pronounced when the earnings news is negative. Both findings are consistent with our prediction that option markets serve as an alternative venue for traders with negative private information seeking to avoid short-sale costs. Additionally, RSS find that the relation between O/S and absolute announcement returns is less pronounced when there is a significant movement in equity prices prior to the announcement date, consistent with informed traders impounding private information into prices ahead of the announcement. In this section, we provide additional evidence that relative option volume reflects private information by examining whether prior week's O/S provides predictive power for the sign and magnitude of quarterly earnings surprises. Our tests build upon RSS by examining the relation between O/S and signed earnings news and returns.

We assemble a new data set from four sources. The OptionMetrics, Compustat Industrial Quarterly, CRSP daily stock, and Institutional Brokers' Estimate System (I/B/E/S) consensus files provide information on option volume, quarterly firm attributes, equity prices, and earnings surprises, respectively. We apply the same sample restrictions outlined in Section 4. The intersection of these four databases results in a final sample consisting of 44,669 firm-quarter observations.

To the extent that informed traders gravitate toward options ahead of negative news, we predict that O/S is negatively associated with the resulting earnings surprise. For each earnings announcement, we measure O/S in the calendar week that directly precedes it. For example, we measure O/S from Monday through Friday of each calendar week and examine the information content of earnings announcements occurring in the subsequent calendar week. This empirical design directly mimics the structure of our main analyses that use O/S in week $w-1$ to predict returns in week w, except here we focus the analysis on the prediction of earnings news and earnings-announcement window returns revealed in week w. In Panel A of Table 7, we use three variables to capture the news released at earnings announcements. The first, SURPRISE, is the earnings surprise as measured by the firm's actual earnings per share (EPS) minus the analyst consensus EPS forecast immediately prior to the announcement, scaled by the beginning-of-quarter stock price. The second, standardized unexplained earnings (SUE), is an alternative measure of earnings surprise defined as the realized EPS minus EPS from four quarters prior, divided by the standard deviation of this difference over the prior eight quarters. The final, CAR$(-1,+1)$, equals three-day cumulative market-adjusted returns during the earnings announcement window from $t-1$ to $t+1$, where day t is the earnings announcement date.

Mirroring the construction of Table 3, Table 7 contains Fama-MacBeth regression results, where standard errors are computed across quarterly coefficients. Panel A demonstrates that the prior calendar week's O/S decile carries predictive power for future earnings surprises. The negative relation between relative option volume and earnings surprises (t-statistic$=-2.36$) is consistent with the negative O/S-return relation reflecting informed trade. We also find analogous results where SUE is the dependent variable. The coefficient on O/S is significantly negative (t-statistic$=-2.06$), indicating that O/S is negatively associated with earnings innovations. The final column of Panel A presents the regression results when the announcement window abnormal returns, CAR$(-1,+1)$, is the dependent variable. The coefficient on O/S remains negative and statistically significant (t-statistic$=-2.11$) incremental to the firm's momentum, size, book-to-market, and decile of equity volume, which is consistent with relative option volume reflecting private information about future asset values revealed in part by the earnings announcement. As an example of the economic significance, the lowest O/S decile outperforms the highest by 0.369% ($=-0.041\times 9$) in the three-day announcement window (all else equal), more than the return spread generated on average during an entire normal week (0.34%).[17]

Panel B of Table 7 examines the predictive power of O/S for returns following the announcement. We use five return windows: CAR$(+2,+5)$, CAR$(+2,+10)$, CAR$(+2,+20)$, CAR$(+2,+40)$, and CAR$(+2,+60)$, where CAR(X,Y), equals the cumulative market-adjusted return from $t+X$ through $t+Y$. The O/S coefficient is insignificant across all of the return horizons, with t-statistics ranging from -0.52 to -1.53. These results indicate that most, if not all, of the private information in O/S in the week prior to the announcement is publicly revealed at the earnings announcement, leaving no significant return predictability in the days following the announcement. Collectively, the results in Table 7 are consistent with O/S reflecting private information about future earnings which is impounded into prices during subsequent earnings announcements.

6. Conclusion

The central contribution of this paper is a mapping between observed transactions and the sign and magnitude of private information that does not require estimating order flow imbalances. Specifically, we examine the information content of option and equity volumes when agents are privately informed but trade direction is unobserved. We provide theoretical and empirical evidence that O/S, the amount of trading volume in option markets relative to equity markets, is a negative cross-sectional signal of private information. Stocks in the

[17] In untabulated results, we find that O/S contains predictive power for earnings news as early as six weeks prior to the earnings announcement, indicating that informed traders' anticipation of earnings news is reflected in the level of O/S several weeks prior to the announcement. Consistent with this interpretation, we find that $\Delta O/S$ and $\Omega O/S$, which rely on weekly changes in O/S, fail to predict earnings announcement news.

Table 7
Earnings surprises and earnings announcement returns.

The sample for Table 7 consists of 44,669 quarterly earnings announcements during the 1996 through 2010 sample window. Each measure of earnings news is regressed on deciles of $O/S_{i,w}$ from the prior calendar week. $O/S_{i,w}$ equals the ratio of option volume to equity volume of firm *i* in week *w*. Deciles range from 1 to 10 with the highest (lowest) values located in the 10th (1st) decile. SURPRISE equals the firm's actual EPS minus the consensus EPS forecasts immediately prior to the announcement, scaled by the beginning-of-quarter share price. SUE equals the standard unexplained earnings, calculated as realized EPS minus EPS from four-quarters prior, divided by its standard deviation over the prior eight quarters. CAR(*X*,*Y*) is the cumulative market-adjusted return from day *X* to day *Y* relative to the earnings announcement. EQVOL equals the total equity volume traded, SIZE is the log of the firm's market capitalization, and LBM is the log of the firm's book-to-market ratio measured at the firm's last quarterly announcement date. MOMEN equals the cumulative market-adjusted returns measured over the six months leading up to portfolio formation, and RET(0) is the cumulative market-adjusted return over the prior month. AMIHUD is the Amihud illiquidity ratio in the week prior to the announcement. All returns are calculated as percentages. Standard errors are computed across quarterly coefficient estimates, following Fama and MacBeth (1973). The resulting *t*-statistics are shown in parentheses. The notations ***, **, and * indicate the coefficient is significant at the 1%, 5%, and 10% level, respectively.

Panel A: Earnings announcement surprises

Dep. variable:	SURPRISE	SUE	CAR(-1,+1)
Intercept	0.096***	0.296***	0.236
	(2.74)	(4.59)	(0.49)
Decile(O/S)	−0.004**	−0.008**	−0.041**
	(−2.36)	(−2.06)	(−2.11)
Decile(ΔEQVOL)	−0.003*	0.006	0.003
	(−1.76)	(1.47)	(0.16)
RET(0)	0.005***	0.005**	−0.063***
	(7.19)	(2.10)	(−5.42)
MOMEN	0.002***	0.007***	0.001
	(9.62)	(11.44)	(0.51)
SIZE	0.001	−0.023***	0.028
	(0.48)	(−3.32)	(0.56)
LBM	−0.167***	−0.822***	0.318
	(−3.98)	(−7.89)	(0.72)
AMIHUD	−0.006***	0.003*	−0.038**
	(−3.96)	(1.67)	(−2.49)
Adj-R^2 (%)	3.525	4.924	1.056

Panel B: Post-earnings-announcement returns

Dep. variable:	CAR(+2,+5)	CAR(+2,+10)	CAR(+2,+20)	CAR(+2,+40)	CAR(+2,+60)
Intercept	−0.897*	−0.773	0.272	0.894	0.734
	(−1.93)	(−0.90)	(0.19)	(0.40)	(0.28)
Decile(O/S)	−0.007	−0.008	−0.020	−0.072	−0.098
	(−0.80)	(−0.52)	(−0.73)	(−1.53)	(−1.39)
Decile(ΔEQVOL)	0.016	0.014	0.019	−0.019	−0.038
	(1.20)	(0.73)	(0.59)	(−0.47)	(−0.88)
RET(0)	0.004	−0.001	0.003	0.013	0.048*
	(0.59)	(−0.09)	(0.15)	(0.57)	(1.65)
MOMEN	−0.005**	−0.009**	−0.013**	−0.003	0.005
	(−2.25)	(−2.49)	(−2.17)	(−0.30)	(0.39)
SIZE	0.080	0.065	−0.014	−0.076	−0.062
	(1.61)	(0.73)	(−0.09)	(−0.32)	(−0.22)
LBM	0.797***	1.151**	1.044	1.066	2.138
	(2.76)	(2.32)	(1.22)	(0.78)	(1.17)
AMIHUD	−0.014	−0.008	−0.020	−0.019	−0.027
	(−1.24)	(−0.53)	(−0.99)	(−0.56)	(−0.73)
Adj-R^2 (%)	2.145	3.232	4.092	5.154	5.836

lowest decile of O/S outperform the highest decile by 0.34% on a factor-adjusted basis in the week following portfolio formation. We offer a simple explanation for this finding, specifically that it results from how informed traders choose between trading in equity and option markets in the presence of short-sale costs.

We model the capital allocation and price-setting processes in a multimarket setting and develop novel predictions regarding information transmission across markets. In equilibrium, short-sale costs cause informed traders to trade more frequently in option markets when in possession of a negative signal than when in possession of a positive signal, thus predicting that volume in options markets, relative to equity markets, is indicative of negative private information. By empirically documenting that O/S is a negative cross-sectional signal for future equity returns, our results are consistent with market frictions preventing equity prices from immediately reflecting the information content of O/S. Return predictability associated with O/S is relatively short-lived, decreasing sharply in the first few weeks and remaining statistically significant in the four weeks following portfolio formation, which suggests that it takes multiple weeks for the information in O/S to become fully reflected in equity prices.

Our model also predicts that O/S is a stronger signal when short-sale costs are high or option leverage is low, and that volume differences across calls and puts predict future return skewness, all of which we confirm in the data. We measure short-sale costs using proprietary firm-specific data on institutional loan fees and loan supply from 2002–2009. We find mixed evidence that O/S alphas increase with short-sale costs and strong evidence that O/S alphas decrease with option leverage. Conditional on low average leverage traded in options, sorting stocks by deciles of O/S results in an average annual hedge return of 27%. Finally, we show that O/S predicts the sign and magnitude of earnings surprises and abnormal returns at quarterly earnings announcements, consistent with O/S reflecting traders' private information.

Appendix A. Simultaneous equations

The full set of simultaneous equations that characterize the equilibrium are

$$a_s = \mu + \frac{\alpha\left(\phi\left(\frac{k_5}{\sigma_\epsilon}\right) - \phi\left(\frac{k_6}{\sigma_\epsilon}\right)\right)}{\alpha\left(\Phi\left(\frac{k_6}{\sigma_\epsilon}\right) - \Phi\left(\frac{k_5}{\sigma_\epsilon}\right)\right) + (1-\alpha)q_1}\sigma_\epsilon, \tag{12}$$

$$b_s = \mu - \frac{\alpha\left(\phi\left(\frac{k_2}{\sigma_\epsilon}\right) - \phi\left(\frac{k_1}{\sigma_\epsilon}\right)\right)}{\alpha\left(\Phi\left(\frac{k_2}{\sigma_\epsilon}\right) - \Phi\left(\frac{k_1}{\sigma_\epsilon}\right)\right) + (1-\alpha)q_2}\sigma_\epsilon, \tag{13}$$

$$a_c = \frac{\alpha\int_{k_6}^{\infty}\phi(\epsilon)C(\epsilon,\sigma_\eta)\,d\epsilon + (1-\alpha)q_3\phi(0)\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}{\alpha\left(1-\Phi\left(\frac{k_6}{\sigma_\epsilon}\right)\right) + (1-\alpha)q_3}, \tag{14}$$

$$b_c = \frac{\alpha\int_{k_2}^{k_3}\phi(\epsilon)C(\epsilon,\sigma_\eta)\,d\epsilon + (1-\alpha)q_4\phi(0)\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}{\alpha\left(\Phi\left(\frac{k_3}{\sigma_\epsilon}\right) - \Phi\left(\frac{k_2}{\sigma_\epsilon}\right)\right) + (1-\alpha)q_4}, \tag{15}$$

$$a_p = \frac{\alpha\int_{-\infty}^{k_1}\phi(\epsilon)P(\epsilon,\sigma_\eta)\,d\epsilon + (1-\alpha)q_5\phi(0)\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}{\alpha\left(\Phi\left(\frac{k_1}{\sigma_\epsilon}\right)\right) + (1-\alpha)q_5}, \tag{16}$$

$$b_p = \frac{\alpha\int_{k_4}^{k_5}\phi(\epsilon)P(\epsilon,\sigma_\eta)\,d\epsilon + (1-\alpha)q_6\phi(0)\sqrt{\sigma_\epsilon^2+\sigma_\eta^2}}{\alpha\left(\Phi\left(\frac{k_5}{\sigma_\epsilon}\right) - \Phi\left(\frac{k_4}{\sigma_\epsilon}\right)\right) + (1-\alpha)q_6}, \tag{17}$$

$$\theta(P(k_1,\sigma_\eta) - a_p) = \gamma(b_s(1-\rho) - \mu - k_1), \tag{18}$$

$$\gamma(b_s(1-\rho) - \mu - k_2) = \theta(b_c - C(k_2,\sigma_\eta)), \tag{19}$$

$$\theta(b_c - C(k_3,\sigma_\eta)) = 0, \tag{20}$$

$$0 = \theta(b_p - P(k_4,\sigma_\eta)), \tag{21}$$

$$\theta(b_p - P(k_5,\sigma_\eta)) = \gamma(\mu + k_5 - a_s), \tag{22}$$

$$\gamma(\mu + k_6 - a_s) = \theta(C(k_6,\sigma_\eta) - a_c). \tag{23}$$

Eqs. (12)–(17) are the zero profit conditions for the market maker. Eq. (12), for example, ensures that the ask price for a stock is exactly the expectation of $\tilde{V}$ conditional on a stock trade. Computing this conditional expectation in the case of options trades (Eqs. (14)–(17)) requires integrating the value function for options C and P. These functions are the mean value of a call and put, respectively, conditional on the signal ϵ and the standard deviation of η. Specifically,

$$C(\epsilon,\sigma_\eta) \equiv \mathrm{E}(\tilde{C}\,|\,\tilde{\epsilon} = \epsilon) = \Phi\left(\frac{\epsilon}{\sigma_\eta}\right)\epsilon + \phi\left(\frac{\epsilon}{\sigma_\eta}\right)\sigma_\eta, \tag{24}$$

$$P(\epsilon,\sigma_\eta) \equiv \mathrm{E}(\tilde{P}\,|\,\tilde{\epsilon} = \epsilon) = -\Phi\left(\frac{-\epsilon}{\sigma_\eta}\right)\epsilon + \phi\left(\frac{\epsilon}{\sigma_\eta}\right)\sigma_\eta. \tag{25}$$

Finally, Eqs. (18) through (23) ensure informed traders are indifferent between the two neighboring portfolios at the cutoff points. For example, (18) ensures they are indifferent between buying puts and shorting stock given the signal k_1.

These equations cannot be solved in closed form due to the nonlinearity of C and P, however, we can prove some general results directly from the simultaneous equations without needing a closed-form solution. Throughout, we assume the exogenous parameters are chosen so that there exists a set of equilibrium parameters satisfying (12) through (23) as well as $k_1 < k_2 < k_3 \le k_4 < k_5 < k_6$. For some parameters, no such equilibrium exists, typically because the informed trader never finds it optimal to trade stock, implying $k_2 = k_3$; we focus on the case when informed traders use stock because our goal is to model the impact of short-sale costs, which are only relevant when informed traders use equity. We do consider parametrizations where the informed trader chooses to trade every signal, meaning $k_3 = k_4$. In this case, we have one fewer free parameter and we need to replace Eqs. (20) and (21) with the single equation:

$$\theta(b_c - C(k_3,\sigma_\eta)) = \theta(b_p - P(k_3,\sigma_\eta)). \tag{26}$$

Appendix B. Measure of leverage

Leverage λ in options markets is measured by the elasticity of the option pricing function $C(S)$ with respect to S. For options priced according to Black-Scholes, we have

$$\lambda = \Phi\left(\frac{\log\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)\frac{S}{C}. \tag{27}$$

The above elasticity represents the change in value of an option position with respect to the change in value of an option position, assuming that because the stock costs S and the option C, the option position has S/C times as many contracts as the stock position has shares. In our model, order sizes are fixed exogenously, so we examine

$\lambda = (\partial C/\partial S)\theta/\gamma$ instead of $\lambda = (\partial C/\partial S)S/C$:

$$\lambda = \frac{\partial C}{\partial S}\frac{\theta}{\gamma} = \frac{\frac{\partial C(\epsilon)}{\partial \epsilon}}{\frac{\partial V}{\partial \epsilon}}\frac{\theta}{\gamma} = \frac{\phi\left(\frac{\epsilon}{\sigma_\eta}\right)\frac{\epsilon}{\sigma_\eta}+\Phi\left(\frac{\epsilon}{\sigma_\eta}\right)+2\sigma_\eta\phi\left(\frac{\epsilon}{\sigma_\eta}\right)}{1}\frac{\theta}{\gamma}. \tag{28}$$

Since the options in our model are struck at μ, we measure λ when $\epsilon = 0$, giving us:

$$\lambda_{JS} = \frac{1}{2}\frac{\theta}{\gamma}. \tag{29}$$

Appendix C. Proofs

Result C.1. When uninformed demand satisfies $q_1 = q_2$ and $q_3 = q_4 = q_5 = q_6$, in equilibrium, $\mathrm{E}(\tilde{V}\,|\,\text{option trade}) \le \mathrm{E}(\tilde{V}\,|\,\text{equity trade})$. We obtain a strict inequality when $\rho > 0$.

Result C.2. Given the same assumptions as Result C.1, the difference in conditional means $D \equiv \mathrm{E}(\tilde{V}\,|\,\text{stock trade}) - \mathrm{E}(\tilde{V}\,|\,\text{option trade})$ is weakly increasing in the short-sale cost ρ.

Proof. Define $\overline{V_O} = \mathrm{E}(\tilde{V}-\mu\,|\,\text{option trade})$ and $\overline{V_S} = \mathrm{E}(\tilde{V}-\mu\,|\,\text{stock trade})$. We show that $D \equiv \overline{V_O} - \overline{V_S}$ is 0 for $\rho = 0$, strictly increasing in ρ at $\rho = 0$, and weakly increasing in ρ at all $\rho > 0$, which together implies both Results 1 and 2.

Given the symmetry in uninformed trader demand and the normal distributions of $\tilde{V}$, $\tilde{\epsilon}$, and $\tilde{\eta}$, when $\rho = 0$ the entire problem is symmetric and therefore, we have $k_1 = -k_6$, $k_2 = -k_5$, $k_3 = -k_4$ in equilibrium. This, in turn, implies that $\overline{V_O} = \overline{V_S} = 0$ when $\rho = 0$.

We first compute $\overline{V_O}$ and $\overline{V_S}$ as a function of the equilibrium cutoff points used by informed traders k_i:

$$\overline{V_S} = \frac{\alpha}{p_S}(\phi(k_1) - \phi(k_2) + \phi(k_5) - \phi(k_6)), \tag{30}$$

$$p_S = (1-\alpha)(q_1+q_2) + \alpha(\Phi(k_2) - \Phi(k_1) + \Phi(k_6) - \Phi(k_5)), \tag{31}$$

$$\overline{V_O} = \frac{\alpha}{p_O}(-\phi(k_1) + \phi(k_2) - \phi(k_3) + \phi(k_4) - \phi(k_5) + \phi(k_6)), \tag{32}$$

$$p_O = (1-\alpha)(q_3+q_4+q_5+q_6) + \alpha(\Phi(k_1) + \Phi(k_3) - \Phi(k_2) + \Phi(k_5) - \Phi(k_4) + 1 - \Phi(k_6)), \tag{33}$$

where p_S and p_O are the unconditional probabilities of a stock trade and an option trade occurring, respectively.

We now consider the derivative of D with respect to ρ:

$$\frac{\partial D}{\partial \rho} = \frac{\overline{V_S}}{\partial \rho} - \frac{\overline{V_O}}{\partial \rho} = \sum_{i=1}^{6}\left(\frac{\overline{V_S}}{\partial k_i} - \frac{\overline{V_O}}{\partial k_i}\right)\frac{\partial k_i}{\partial \rho}. \tag{34}$$

The derivatives $\partial k_i/\partial \rho$ represent changes in equilibrium k_i as ρ changes. The direct effect of ρ on k_i is that, given unchanged prices, selling stock becomes less profitable than it was before. Of course, given the direct effect on k_i, there is also the indirect effect that comes through prices: informed traders' strategy changes, which changes prices, which in turn changes informed traders' strategy. However, due to uninformed traders' demand, these indirect effects dampen the direct effect but do not change its direction. We therefore focus on the first-order change in k_i with respect to ρ.

Since ρ does not appear in the indifference equations at k_3, k_4, k_5, and k_6 (Eqs. (20)–(23)), we have $\partial k_i/\partial \rho = 0$ for all $i \ge 3$. For k_1, we work from the informed traders' indifference Eq. (18):

$$\gamma(b_s(1-\rho) - \mu - k_1) = \theta(P(k_1,\sigma_\eta) - a_p)$$

$$\Rightarrow -b_s - \frac{\partial k_1}{\partial \rho} = \frac{\theta}{\gamma}\frac{\partial P}{\partial k}\frac{\partial k_1}{\partial \rho} = -\frac{\theta}{\gamma}\Phi\left(\frac{-k_1}{\sigma_\eta}\right)\frac{\partial k_1}{\partial \rho}$$

$$\Rightarrow \frac{\partial k_1}{\partial \rho} = -\frac{b_s}{1 - \frac{\theta}{\gamma}\Phi\left(\frac{-k_1}{\sigma_\eta}\right)}. \tag{35}$$

Since $\theta/\gamma \ge 2$ by assumption, and $\Phi(-k_1/\sigma_\eta) > 0.5$ because $k_1 < 0$, we have $\partial k_1/\partial \rho > 0$.

A similar calculation yields

$$\frac{\partial k_2}{\partial \rho} = -\frac{b_s}{1 - \frac{\theta}{\gamma}\Phi\left(\frac{k_2}{\sigma_\eta}\right)}. \tag{36}$$

In order to sign $\partial k_2/\partial \rho$, we note that for signals slightly less than k_2, the informed trader prefers selling stock, while for signals slightly more than k_2, the informed trader prefers selling calls. This implies:

$$\frac{\partial \text{Profit from selling calls}}{\partial k}(k_2) > \frac{\partial \text{Profit from selling stock}}{\partial k}(k_2)$$

$$\Rightarrow \frac{\partial}{\partial k_2}\theta(b_c - C(k_2,\sigma_\eta)) > \frac{\partial}{\partial k_2}\gamma(b_s(1-\rho) - \mu - k_2)$$

$$\Rightarrow -\theta\Phi\left(\frac{k_2}{\sigma_\eta}\right) > -\gamma \Rightarrow \frac{\theta}{\gamma}\Phi\left(\frac{k_2}{\sigma_\eta}\right) < 1 \Rightarrow \frac{\partial k_2}{\partial \rho} < 0. \tag{37}$$

From Eqs. (30)–(33), remembering that p_S and p_O are functions of k_i, we compute:

$$\frac{\partial \overline{V_S}}{\partial k_1} = \alpha\frac{\phi(k_1)}{p_S}(\overline{V_S} - k_1), \tag{38}$$

$$\frac{\partial \overline{V_S}}{\partial k_2} = \alpha\frac{\phi(k_2)}{p_S}(k_2 - \overline{V_S}), \tag{39}$$

$$\frac{\partial \overline{V_O}}{\partial k_1} = \alpha\frac{\phi(k_1)}{p_O}(k_1 - \overline{V_O}), \tag{40}$$

$$\frac{\partial \overline{V_O}}{\partial k_2} = \alpha\frac{\phi(k_2)}{p_O}(\overline{V_O} - k_2). \tag{41}$$

As discussed above, $\overline{V_S} = \overline{V_O} = 0$ when $\rho = 0$. In this case, since $k_1 < k_2 < 0$, it is clear from (38)–(41) that $\partial\overline{V_S}/\partial k_1 > 0$, $\partial\overline{V_S}/\partial k_2 < 0$, $\partial\overline{V_O}/\partial k_1 < 0$, and $\partial\overline{V_O}/\partial k_2 > 0$. Furthermore, since these derivatives are all zero whenever $\overline{V_S} > -k_1$, $k_2 > \overline{V_S}$, $k_1 > \overline{V_O}$, and $\overline{V_O} - k_2$, respectively, the derivatives can never change signs. For example, as $\overline{V_S}$ approaches k_1, the derivative of $\overline{V_S}$ approaches zero, meaning it stops changing and never crosses k_1. Similar logic applies to the other three derivatives, meaning that their sign when $\rho = 0$ applies for all ρ.

Returning to Eq. (34), we have

$$\frac{\partial D}{\partial \rho} = \left(\underbrace{\frac{\partial \overline{V_S}}{\partial k_1}}_{>0} - \underbrace{\frac{\partial \overline{V_O}}{\partial k_1}}_{<0} \right) \underbrace{\frac{\partial k_1}{\partial \rho}}_{>0} + \left(\underbrace{\frac{\partial \overline{V_S}}{\partial k_2}}_{<0} - \underbrace{\frac{\partial \overline{V_O}}{\partial k_2}}_{>0} \right) \underbrace{\frac{\partial k_2}{\partial \rho}}_{<0} \Rightarrow \frac{\partial D}{\partial \rho} > 0. \tag{42}$$

We have a strict inequality here because we assumed $k_1 < k_2$, meaning the informed trader shorts stocks for a non-empty set of signals. If the short-sale costs are sufficiently high that $k_1 = k_2$, further increases no longer have any impact on equilibrium and $\Rightarrow \partial D/\partial \rho = 0$. □

Result C.3. The difference in conditional means $D \equiv \mathrm{E}(\tilde{V}\,|\,\text{option trade}) - \mathrm{E}(\tilde{V}\,|\,\text{equity trade})$ *is decreasing in the leverage in options as measured by* $\lambda = \theta/2\gamma$.

Proof. Since any solution to the simultaneous equations (12)–(23) for order sizes (γ,θ) is also a solution for order sizes $(c\gamma, c\theta)$ for all constants c, we assume without loss of generality that $\gamma = 1$. Following the notation from the proof of Result C.1, we therefore want to show:

$$\frac{\partial D}{\partial \theta} = \sum_{i=1}^{6} \left(\frac{\overline{V_S}}{\partial k_i} - \frac{\overline{V_O}}{\partial k_i} \right) \frac{\partial k_i}{\partial \theta} < 0$$

whenever $\rho > 0$.

Since θ cancels out in the equations for k_3 and k_4, we have

$$\frac{\partial D}{\partial \theta} = \left(\frac{\overline{V_S}}{\partial k_1} - \frac{\overline{V_O}}{\partial k_1} \right) \frac{\partial k_1}{\partial \theta} + \left(\frac{\overline{V_S}}{\partial k_2} - \frac{\overline{V_O}}{\partial k_2} \right) \frac{\partial k_2}{\partial \theta} + \left(\frac{\overline{V_S}}{\partial k_5} - \frac{\overline{V_O}}{\partial k_5} \right) \frac{\partial k_5}{\partial \theta} + \left(\frac{\overline{V_S}}{\partial k_6} - \frac{\overline{V_O}}{\partial k_6} \right) \frac{\partial k_6}{\partial \theta}. \tag{43}$$

We first focus on the partial derivatives $\partial k_i/\partial \theta$. Following the methodology used to compute $\partial k_i/\partial \rho$ in Result C.1, we find

$$\frac{\partial k_1}{\partial \theta} = \frac{P(k_1,\sigma_\eta) - a_p}{\theta \Phi\left(\frac{-k_1}{\sigma_\eta}\right) - 1} > 0, \tag{44}$$

$$\frac{\partial k_2}{\partial \theta} = \frac{b_c - C(k_2,\sigma_\eta)}{\theta \Phi\left(\frac{k_2}{\sigma_\eta}\right) - 1} < 0, \tag{45}$$

$$\frac{\partial k_5}{\partial \theta} = \frac{b_p - P(k_5,\sigma_\eta)}{1 - \theta \Phi\left(\frac{-k_5}{\sigma_\eta}\right)} > 0, \tag{46}$$

$$\frac{\partial k_6}{\partial \theta} = \frac{C(k_6,\sigma_\eta) - a_c}{1 - \theta \Phi\left(\frac{k_6}{\sigma_\eta}\right)} < 0. \tag{47}$$

The logic in Result C.1 implies that $b_p > b_c$, $-k_2 > k_5$, and $-k_1 < k_6$ whenever $\rho > 0$. These facts, along with $C(k,\sigma_\eta) = P(-k,\sigma_\eta)$, imply that

$$-\frac{\partial k_2}{\partial \theta} = \frac{b_c - C(k_2,\sigma_\eta)}{1 - \theta \Phi\left(\frac{k_2}{\sigma_\eta}\right)} < \frac{b_p - C(k_2,\sigma_\eta)}{1 - \theta \Phi\left(\frac{k_2}{\sigma_\eta}\right)} = \left.\frac{\partial k_5}{\partial \theta}\right|_{k_5 = -k_2} < \frac{\partial k_5}{\partial \theta}, \tag{48}$$

$$\frac{\partial k_1}{\partial \theta} = \frac{P(k_1,\sigma_\eta) - a_p}{\theta \Phi\left(\frac{-k_1}{\sigma_\eta}\right) - 1} < \frac{P(k_1,\sigma_\eta) - a_c}{\theta \Phi\left(\frac{-k_1}{\sigma_\eta}\right) - 1} = -\left.\frac{\partial k_6}{\partial \theta}\right|_{k_6 = -k_1} < -\frac{\partial k_6}{\partial \theta}. \tag{49}$$

Adding together (48) and (49) and switching signs yields $(\partial/\partial\theta)(k_2 - k_1) > (\partial/\partial\theta)(k_6 - k_5)$. Since both sides are negative, this implies that the "short stock" region $k_2 - k_1$ shrinks as θ increases, but not as fast as the "long stock" region $k_6 - k_5$ shrinks, implying that the sum in (43) is negative. □

Result C.4. Equity value has a higher skewness conditional on a call trade than conditional on a put trade when $q_i > \alpha/(1-\alpha)139.2$.

Proof. We show that the third centralized moments conditional on call and put trades satisfy

$$\mathrm{E}((\tilde{V} - \hat{V}_{\text{call}})^3\,|\,\text{call trade}) > 0 > \mathrm{E}((\tilde{V} - \hat{V}_{\text{put}})^3\,|\,\text{put trade}), \tag{50}$$

where $\hat{V}_i$ is the expected value of $\tilde{V}$ conditional on trade type i. Inequality (50) implies Result C.4 because skewness is the third centralized moment scaled by a positive number.

We show here that $\mathrm{E}((\tilde{V} - \hat{V}_{\text{call}})^3\,|\,\text{call trade}) > 0$. The other half of inequality (50) follows from the same derivation applied to the put option.

To simplify notation, we write $\mathrm{E}^C(\cdot)$ as short-hand for $\mathrm{E}(\cdot\,|\,\text{call trade})$, and cm_3^C for the third centralized moment conditional on a call trade:

$$cm_3^C = \mathrm{E}^C((\tilde{V} - \hat{V}_{\text{call}})^3) = \mathrm{E}^C((\tilde{\epsilon} - E^C(\tilde{\epsilon}) + \tilde{\eta})^3). \tag{51}$$

Since $\tilde{\epsilon} - \mathrm{E}^C(\tilde{\epsilon})$ and $\tilde{\eta}$ are independent and both have zero mean conditional on a call trade, we have

$$cm_3^C = \mathrm{E}^C((\tilde{\epsilon} - \mathrm{E}^C(\tilde{\epsilon}))^3) \Rightarrow cm_3^C \propto \mathrm{E}^C((\tilde{\delta} - \mathrm{E}^C(\tilde{\delta}))^3), \tag{52}$$

where $\tilde{\delta} = \tilde{\epsilon}/\sigma_\epsilon$ and $\propto$ indicates that the two expressions have the same sign.

Next we break up the expectation in (52) into two exhaustive cases: the trade was initiated by an informed trader and the trade was initiated by an uninformed trader. In each case, we expand $(\tilde{\delta} - \mathrm{E}^C(\tilde{\delta}))^3$, and in order to keep the expression as brief as possible, we write

$$m_i^I \equiv \mathrm{E}(\tilde{\epsilon}^i\,|\,\text{informed call trade}), \tag{53}$$

$$m_i^U \equiv \mathrm{E}(\tilde{\epsilon}^i\,|\,\text{uninformed call trade}), \tag{54}$$

$$\hat{\delta} \equiv \mathrm{E}^C(\tilde{\delta}), \tag{55}$$

$$\alpha_C \equiv P(\text{informed}\,|\,\text{call trade}). \tag{56}$$

After breaking up and expanding the expectation, we find

$$cm_3^C \propto \alpha_C(m_3^I - 3m_2^I\hat{\delta} + 3m_1^I\hat{\delta}^2 - \hat{\delta}^3) + (1-\alpha_C)(m_3^U - 3m_2^U\hat{\delta} + 3m_1^U\hat{\delta}^2 - \hat{\delta}^3)$$

$$=\alpha_C(m_3^I-3m_2^I\hat{\delta}+3m_1^I\hat{\delta}^2-\hat{\delta}^3)+(1-\alpha_C)(-3\hat{\delta}-\hat{\delta}^3)$$
$$=m_3^I\alpha_C+2\hat{\delta}^3-3\hat{\delta}(1+\alpha_C(m_2^I-1)). \quad (57)$$

To arrive at Eq. (57) we use the fact that $\hat{\delta}=\alpha_C m_1^I+(1-\alpha_C)m_1^U=\alpha_C m_1^I$.

From here, we prove three lemmas that together complete the proof under the following condition:

$$q_i>\frac{\alpha}{(1-\alpha)139.2}. \quad (58)$$

This condition ensures that the number of uninformed traders in options markets does not approach zero, in which case markets begin to fail and the skewness result can reverse. It is a condition easily satisfied for any normal parametrizations. If $\alpha>\frac{1}{10}$, we only require $q_i>\frac{1}{1250}$ and if $q_i>\frac{1}{84}$, we only require $\alpha<63\%$.

Lemma 1 shows that $m_3^I>0$ when (58) holds. Lemma 2 shows that $2\hat{\delta}^3-3\hat{\delta}(1+\alpha_C(m_2^I-1))>0$ when $\delta<0$. Lemma 3 shows that $m_3^I\alpha_C>-2\hat{\delta}^3+3\hat{\delta}(1+\alpha_C(m_2^I-1))$ when $\delta>0$ and (58) holds. Put together with (57), these lemmas complete the proof. □

Lemma 1. The third moment of $\tilde{\delta}$ conditional on an informed call trade, m_3^I, is positive whenever $q_i>\alpha/(1-\alpha)139.2$.

Proof. The lemma follows from informed traders' equilibrium cutoff strategy, which assures that a call trade is either weakly bad news or extremely good news. We only need to rule out the possibility that uninformed traders are so scarce that the informed trader almost never buys calls, which would make the distribution of $\tilde{\delta}$ conditional on an informed trade similar to the distribution of $\tilde{\delta}$ conditional on a call *sell*, which has a negative third moment.

From the moments of the truncated normal distribution given in Jawitz (2004), we have

$$m_3^I=\frac{(j_2^2+2)\phi(j_2)-(j_3^2+2)\phi(j_3)+(j_6^2+2)\phi(j_6)}{\Phi(j_3)-\Phi(j_2)+1-\Phi(j_6)}, \quad (59)$$

where j_i are the equilibrium cutoff points scaled down by σ_ϵ so they are $\tilde{\delta}$ cutoffs rather than $\tilde{\epsilon}$ cutoffs. The function $f(x)=(x^2+2)\phi(x)$ is positive, symmetric about $x=0$, decreasing for $x>0$, increasing for $x<0$, and satisfies $f(-\bar{j})+f(\bar{j})=f(0)$ for $\bar{j}=1.832$. In equilibrium, we know that $j_2\le j_3\le 0\le j_6$ and $|j_3|<|j_2|<|j_6|$, so (59) tells us that $m_3^I>0$ whenever $j_6<\bar{j}$.

Next we show that $j_6<\bar{j}$ whenever (58) holds. Assume the contrary, that $j_6\ge\bar{j}$. We consider only equilibria where the informed trader buys equity for some signals, so we know that at $\tilde{\epsilon}=\bar{j}\sigma_\epsilon$ the informed trader prefers equity to calls. Writing $C(x,\sigma_\eta)$ for $E(\tilde{C}|\tilde{\epsilon}=x)$, we have that

$$j_6\ge\bar{j}\Rightarrow\gamma(\mu+\sigma_\epsilon\bar{j}-a_s)>\theta(C(\bar{j}\sigma_\epsilon,\sigma_\eta)-a_c). \quad (60)$$

The right-hand side of (60) is increasing in σ_η, so if (60) holds when $\sigma_\eta=0$, it holds for all σ_η.

When $\sigma_\eta=0$, we can solve for the equilibrium k_6 directly from the simultaneous equations in Appendix A. In particular, we find that

$$k_6=\frac{\mu-a_s+\theta a_c}{\theta-1}. \quad (61)$$

So if $k_6\ge\bar{j}\sigma_\epsilon$, we have

$$\bar{j}\sigma_\epsilon\ge\frac{\mu-a_s+\theta a_c}{\theta-1}\Rightarrow\bar{j}\sigma_\epsilon\ge a_c\Rightarrow\bar{j}\sigma_\epsilon\ge\frac{\alpha\phi(\bar{j})}{\alpha(1-\Phi(\bar{j}))+(1-\alpha)q_6}\sigma_\epsilon. \quad (62)$$

Solving (62) for q_6, we find exactly the opposite of the condition (58), so we know that (58) implies $k_6<\bar{j}\sigma_\epsilon$ and $m_3^I>0$. □

Lemma 2. When $\delta<0$, we have that $2\hat{\delta}^3-3\hat{\delta}(1+\alpha_C(m_2^I-1))>0$.

Proof. This lemma holds because the quantity in question measures the difference between non-centralized moments and centralized moments due to the change in mean. The lemma shows that when the mean of a variable is negative, the centralized third moment is greater than the non-centralized third moment. To see this technically, first note that

$$\text{var}(\tilde{\delta}|\text{call trade})=E^C(\tilde{\delta}^2)-\hat{\delta}^2=\alpha_C m_2^I+(1-\alpha_C)-\hat{\delta}^2$$
$$=1+\alpha_C(m_2^I-1)-\hat{\delta}^2. \quad (63)$$

And since variances are positive, we have

$$1+\alpha_C(m_2^I-1)-\hat{\delta}^2>0\Rightarrow\hat{\delta}(1+\alpha_C(m_2^I-1))<\hat{\delta}^3$$
$$\Rightarrow 2\hat{\delta}^3-3\hat{\delta}(1+\alpha_C(m_2^I-1))>0. \quad \square \quad (64)$$

Lemma 3. When $\delta>0$ and (58) holds, we have that $m_3^I\alpha_C>-2\hat{\delta}^3+3\hat{\delta}(1+\alpha_C(m_2^I-1))$.

Proof. The intuition for Lemma 3 is that when $\hat{\delta}>0$, the centralized third moment is less than the non-centralized third moment, but the positive mean makes the third moment so large it is positive even after centralization. More rigorously, we have

$$m_3^I\alpha_C+2\hat{\delta}^3-3\hat{\delta}(1+\alpha_C(m_2^I-1))$$
$$\propto m_3^I+2(m_1^I)^3(\alpha_C)^2-3m_1^I(1+\alpha_C(m_2^I-1))$$
$$>m_3^I-3m_1^I(1+\alpha_C(m_2^I-1)). \quad (65)$$

From Jawitz (2004), we have

$$m_3^I=\frac{(j_2^2+2)\phi(j_2)-(j_3^2+2)\phi(j_3)+(j_6^2+2)\phi(j_6)}{\Phi(j_3)-\Phi(j_2)+1-\Phi(j_6)}, \quad (66)$$

$$m_2^I=\frac{(j_2)\phi(j_2)-(j_3)\phi(j_3)+(j_6)\phi(j_6)}{\Phi(j_3)-\Phi(j_2)+1-\Phi(j_6)}, \quad (67)$$

$$m_1^I=\frac{\phi(j_2)-\phi(j_3)+\phi(j_6)}{\Phi(j_3)-\Phi(j_2)+1-\Phi(j_6)}. \quad (68)$$

Noting that any equilibrium satisfying (58) and $\hat{\delta}>0$ in which the informed trader uses each asset with positive probability satisfies:

1. $-\bar{j}<j_2<j_3<0<j_6<\bar{j}$.
2. $|j_3|<|j_2|<|j_6|$.
3. $\phi(j_2)-\phi(j_3)+\phi(j_6)>0$.

We can substitute these conditions into (65) and find that $m_3^I-3m_1^I(1+\alpha_C(m_2^I-1))>0$, which in turn implies Lemma 3. □

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